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Selection and Performance of Vibration Tests

Allen J. Curtis
Nickolas G. Tinling
Henry T. Abstein, Jr.

Hughes Aircraft Company

1971

The Shock and Vibration Information Center
United States Department of Defense

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THE SHOCK AND VIBRATION INFORMATION CENTER
Naval Research Laboratory
Washington, D.C.

William W. Mutch, *Head*

Henry C. Pusey

Rudolph H. Volin

Edward H. Schell

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FOREWORD

Vibration testing is a rapidly evolving technology that has gained widespread recognition of its importance mostly in the period since World War II. As often happens in a new and rapid technological development, vibration testing has been beset by unclear, conflicting, and sometimes controversial concepts of test specifications, test conditions, test methods, and interpretation of test results.

In this monograph, the authors have done a great service by compiling the state-of-the-art knowledge of vibration testing and related technology in a very clear, concise, and comprehensive manner. The various test methods are described and explained in a factual way so that the reader can easily assimilate the essential concepts and then use his own engineering judgment in applying them to the problem at hand. Nevertheless, the authors do not hesitate to express their own opinions and judgments in a scientific manner, and these provide authoritative precepts which can be very helpful in guiding the practitioner.

The designer and specification writer will find helpful explanations and background in Chapter 2, "Selection of Appropriate Test Method." These people often are not vibration specialists, but this chapter will help them acquire an understanding that will increase their effectiveness in incorporating suitable provisions for vibration in their designs and specifications.

Chapter 3, "Simulation Characteristics of Test Methods," will be especially useful to those who have had difficulty in understanding the basic features of the various vibration test methods. This chapter clearly delineates each method and with a minimal amount of mathematics summarizes the analytical basis of each.

The test conductor will be concerned primarily with Chapters 4 and 5. Much of the practical information needed to conduct a vibration test is given in these chapters, including many helpful hints which have been acquired through the experience and mistakes of others.

The authors conclude with a summary in Chapter 6 of how vibration data are acquired and handled. And so this monograph gives the vibration testing technologist a reference that systematically reviews and explains how vibration data are acquired, how the data are used in preparing specifications, how a test is conducted to satisfy specifications, and how test results are interpreted.

This monograph should help those working with vibration problems, and particularly the novice, to come to a clearer understanding of the basis, concepts, and purposes of vibration testing, and it will be much appreciated by those who desire to see how the technology fits together to make good sense.

D.C. KENNARD, JR.

Test and Evaluation Division
Goddard Space Flight Center
Greenbelt, Maryland

PREFACE

The continuing development of the field of vibration testing is evidenced by the number of technical meetings and extensive literature devoted to the topic. Preparation of a monograph which will adequately describe the selection and performance of vibration tests and which will not rapidly become obsolescent is perhaps impossible. Nevertheless, the authors believed that one of the series of monographs sponsored by Shock and Vibration Information Center should be addressed to this subject. It seemed that a wealth of analytical, empirical, and practical information was scattered in various technical journals, government and contractor reports, and perhaps mainly in the subconsciousness of many workers in the field. They also felt that a document that gathered, sifted, and collated this information would prove instructive to newcomers to the field and useful as a reference for the "old hands." To avoid the problem of obsolescence insofar as possible, the monograph should be restricted to facts and principles which can be used to make sound engineering decisions, and thus should be relatively independent of future developments of test techniques and vibration test equipment.

The monograph which has resulted from these ideas was made possible primarily by the support of Dr. W. W. Mutch and his staff at the Shock and Vibration Information Center. The authors must also acknowledge the contribution of those reviewers of an earlier draft whose generous and constructive comments added much to the final version.

It would have been impossible to write this monograph without a rather extended association with a vibration test laboratory and some initial inspiration to pursue endeavours in the field. The opportunity to participate for a number of years in the development of new, and hopefully improved, simulation techniques while associated with the environmental test laboratory at Hughes Aircraft Company is gratefully acknowledged. Finally, the two more senior authors would like to thank Dr. C. T. Morrow whose initial pioneering of random vibration testing at Hughes provided the spark for their continuing interest and activity in the field of vibration testing.

ALLEN J. CURTIS
NICHOLAS G. TINLING
Los Angeles, California HENRY T. ABSTEIN, JR.

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CHAPTER 1
INTRODUCTION

According to Webster, a monograph is a written account of a single thing or class. Alternatively, and perhaps synonomously, it is a special treatise on a particular subject. Again, from the same source, a treatise is a methodical discussion of the facts and principles involved and conclusions reached. In one sense, then, the following chapters may not constitute a monograph since, as the title proclaims, both the selection and the performance of vibration tests are to be treated. However, the selection of a test without knowledge of how or whether it can be performed, or conversely, the performance of a test without knowing why it was selected, is sterile indeed. Thus, in a larger sense, the selection and performance of vibration tests are appropriate subjects for one monograph. It is intended to be a treatise, as defined above, within the limitations of the scope of the subject matter described below and with a further reservation that the reader will be expected to draw the final conclusions, or make the final engineering decisions, based on the facts and principles discussed.

1.1 Purpose

A first exposure to the performance of a vibration test can be a bewildering experience, with the observer very unsure of what happened and even less sure why. If this experience arouses enough interest to visit the library, that observer is likely to find a number of textbooks which hardly mention vibration testing and a larger number of journal articles which generally assume the reader already knows all the principles of vibration testing from reading the textbooks. The primary purpose of this monograph is to fill that gap by presenting a methodical discussion of the facts and principles to be applied to the selection and performance of vibration tests which will be of value as a reference document to both the newly involved and the experienced worker in the field.

It is intended that this discussion be presented at a technical level which strikes a middle ground between the elementary discussion of simple dynamic systems found in the first chapters of vibration textbooks (which should already be familiar to the reader) and the more detailed discussions of complex dynamic systems, noise theory, and electronic equipment, which are generally unnecessary to an understanding of the basic parameters. Thus a reader who has mastered the equivalent of a first course in mechanical vibrations, either formally in the classroom or by experience, and who has a minimal familiarity with the conduct of a vibration test should expect this monograph to be quite readable. Where appropriate, brief and simplified discussions of theory are included along with citations of more complete and rigorous presentations.

1.2 Scope

It has been indicated that the subject matter will be limited to principles and facts, supplemented by citations of significant contributions to the field. By implication, then, the scope of the monograph excludes detailed descriptions of the operation and construction of particular equipment and, instead, describes the functions and purposes of required equipment without reference to any commercial models.

The expression *vibration tests* may have a number of different connotations. Within the scope of this monograph, vibration tests are those tests where a physical object, from a fraction of an ounce to many tons in weight, is subjected to a controlled external vibratory excitation. Thus a wide variety of vibration tests in which internally generated excitations cause vibration of a physical object are excluded. For instance, the testing of an antenna under the vibration loads due to its mechanical scanning and the testing of turbine blading under the vibratory loads due to gas flow during operation are excluded.

External vibratory excitation is applied to physical objects for a number of reasons, not all of which would be classified as tests. For example, vibration excitation is used in the materials handling field for packing crates, unloading hopper-type vehicles, etc. Thus vibration tests in this context are limited to those situations where external excitation is applied to an object in order to determine the manner in which the object physically or functionally responds to that excitation and to determine any effects it may have on the object.

There are three basic methods of applying external vibratory excitation to a physical object. First, and most commonly, by application of sufficient but undefined force at one or more discrete points of the object to create a desired motion. Second, by application of a desired force at one or more discrete points of the object. Third, the object may be "immersed" in a desired acoustic field, the pressure fluctuations of which constitute a force excitation over the entire surface of the object. Four generic types of tests, which are identified by these three methods are (a) motion testing; (b) force-control testing; (c) acoustic testing; and (d) impedance testing, which is a combination of motion and force-control testing. However, for a number of reasons, acoustic testing is generally considered distinct from vibration testing and will not be included.

Any discussion of problems and limitations of vibration testing among workers in the field will almost certainly get around to one of two topics, mechanical impedance effects or vibration equivalence. Both topics will necessarily enter into this monograph. However, only material on these topics which is incidental to the main discussion will be included. Descriptions of impedance test methods and equipment will not be included.

Finally, it is not intended that this monograph serve as a manual to be used in either selecting or performing a particular test under a particular set of circumstances. Rather, the intention is to provide the reader with sufficient information to permit him, for his particular circumstances, to make a logical selection

of test methods and to ensure that, once selected, the test is conducted in a proper manner.

To fulfill this intention, the succeeding chapters are structured as follows. Chapter 2 contains a discussion of the numerous factors which must be considered in the selection of the appropriate test method. First, a number of general considerations, such as defining the purpose of the test, are described, followed by discussions of the selection of test conditions and procedures. The next chapter includes a detailed technical description of the various test methods, e.g., sinusoidal, random, etc., together with a discussion of what can be achieved by their use, i.e., the simulation characteristics of each method. This chapter is followed by chapters devoted to vibration equipment requirements, performance and control of the various methods, and the acquisition and analysis of vibration data obtained during the tests.

1.3 Necessity of Vibration Tests

It would be interesting to arrange a poll of engineers involved in vibration testing, in one capacity or another, which posed the question: "Why is this test being performed?" The percentage of answers which would be variations of either, "I don't know," or "That's what the spec says," would probably be distressingly large. A later section will discuss the purpose of vibration tests in detail. It is appropriate to consider here the general purposes of vibration testing.

In one way or another, almost all vibration tests, as defined previously, are employed to ensure the suitability of the test object for its intended use. The rather wide variety of tests which can be and are performed grew out of the wide variety of intended uses, i.e., environments, and the differing criteria for establishing suitability. However, it is believed that all tests are intended to establish suitability with respect to at least one of the following three criteria: (a) structural integrity; (b) adequate functional performance; and (c) quality assurance level; i.e., adequate workmanship. There are probably those who would state that a fourth criterion of adequate equipment reliability should be added. However, it would seem that criteria a and c together encompass the purposes of reliability testing.

Given that vibration tests have the above-mentioned purposes, the question still remains - why are vibration tests necessary? Is it as Tetyve exclaims in *Fiddler on the Roof*, "Tradition!"? Certainly this is a factor, though perhaps more in the selection of tests than in the question of whether to test. A more rational reason is that they are performed to save money and, in a number of cases, lives, by uncovering design or construction weaknesses that would cause failure due to the vibration encountered in usage. By simulating either the usage vibration itself or its effects, these weaknesses can be uncovered in the laboratory quite economically compared to the cost of occurrence during use. Furthermore, the behavior of the test object can be observed and carefully measured with instrumentation in much greater detail than is generally possible in the

usage environment, particularly with "one-shot" devices such as missiles and space vehicles.

1.4 Historical Development

The general subject of mechanical vibration is itself a rather young specialty, since the first college course devoted to the subject was introduced as recently as 1928. The need for such a specialized course in applied mechanics arose because of the development of higher speed and higher powered rotating machinery, such as automobile engines, steam turbines, and electric generators. Unbalance in the moving parts of these machines produced vibrations, mainly at the operating speed (or frequency), although harmonics of this fundamental frequency also occurred. Thus the motions were essentially periodic, and Fourier series could be used both to analyze and to compute the response to these vibrations. With the development of propeller-driven aircraft, these same techniques could be extended to cope with the vibration environments for aircraft and airborne equipment. Concurrently with this development cycle, the development of vibration testing equipment (or "shakers") took the course of mechanically driven machines using eccentric drives, followed by the electrodynamic or "loudspeaker" type of equipment driven by variable-speed motor-generator sets.

With the development, mainly since World War II, of jet engines, rocket motors, aircraft, and missiles whose performance is high enough to require analyses of the effects of turbulent boundary layer, etc., the vibrations to be dealt with were found to be no longer adequately described by the simple periodic motion/Fourier series approach. Instead, it was found that the vibrations (or the excitation forces which produced them) could, in many cases, only be described in statistical terms since the amplitudes were found to fluctuate in a random manner. Several developments were required to make effective use of this new approach. First, the development of improved electronic instrumentation systems was required. Particularly significant was the development of magnetic tape recording systems which permitted the repeated reproduction of an electrical voltage proportional to the measured vibration (displacement, velocity, etc.). Second, more sophisticated equipment for data reduction, again electronic in the main, was developed to handle the more complex procedures required. The third factor which made the use of the concept of random vibration possible was the development of high-output power amplifiers to drive the electrodynamic shakers in accordance with almost any desired input signal. For example, actual flight vibration measurement recordings have been played into power-amplifier/shaker systems.

During these development cycles, it was fortunate that much of the mathematical theory required had already been developed. In the field of communications, the problem of electrical noise in circuitry had received a great deal of attention, and it turned out that the theory and analysis techniques which evolved were almost directly applicable to the analysis of random vibration.

With the means of analyzing properly a complex vibration signal, of synthesizing the desired excitation signal through spectrum shaping networks, and of driving a shaker in accordance with this excitation, the one remaining development which enabled the complexity and sophistication of present testing methods was the application of servomechanism techniques, i.e., automatic gain control, to the control of vibration test level. Two of the authors can remember, all too clearly, when two test items were needed for each random vibration test. One was used and virtually destroyed during the lengthy manual equalization of the test spectrum, whereas the second, when available, was used to fulfill the purposes of the test. This situation did not make the customers too happy or create the impression that the test engineers knew quite what they were about. With servomechanism control, now known as automatic equalization, random vibration tests became economical, efficient, and generally acceptable.

Until the late 1960's, almost all activities leading to the establishment or conduct of vibration tests were carried out using analog information processing. Now, however, digital processing, with the speed and accuracy which it can provide, is replacing analog methods more and more, most recently with the introduction in 1969 of digital synthesis and analysis of the vibration test signal.

Thus the state of the art in vibration testing, and "art" is used advisedly, has progressed very rapidly during the last two decades. However, it is expected that the basic principles described in the following chapters will, as principles should, remain valid even when unknown future developments take their historical place.

CHAPTER 2
SELECTION OF APPROPRIATE TEST METHOD

It is an infrequent occasion when a reader, by himself, will have the opportunity to make an optimum selection of a vibration test based on adequate technical information about the test object and the overall purpose of the test. In the first place, later discussions will illustrate that technical information is often insufficient at the time of test selection. Second, few readers will be in a position where they alone can make the selection. Third, the influence of a number of nontechnical factors may override much of the technical consideration. These are factors such as schedule, cost, tradition, hardware availability, etc. Thus the material to be discussed in this chapter must necessarily be presented somewhat idealistically. However, this discussion should also find applicability on those occasions when modification of an existing test program is required and when assessment of results at the conclusion of a test program is undertaken.

The following sections discuss the numerous interrelated factors which should be considered in the test selection. Although interrelated, the discussion of each factor is necessarily independent with the interrelationships being either indicated or self-evident. The factors have been grouped into general considerations, test conditions, data requirements, and necessary accuracy.

2.1 General Considerations

Test Purpose

It should be superfluous to say that the first consideration in the selection of a test should be definition of the purpose of the test. Unfortunately, it is the authors' experience that such is not always the case and that, on occasion, eventual consideration of the purpose has led to deletion of the test, either because there was no real purpose or because the purpose could not be achieved by conduct of a feasible vibration test. In many cases, the purpose of the test is explicit in the label applied to the test program, e.g., qualification test or flight acceptance test. In other cases, the purpose is implicit in the type of hardware under test, e.g., developmental or production equipment. Another implicit definition of test purpose is indicated by the assembly level of the test object, i.e., a single-piece part or a complete spacecraft. It was postulated in Section 1.3 that all tests either should or do have the basic purpose of establishing suitability for the intended use with respect to either structural integrity, functional performance, or workmanship. A more detailed classification of basic purpose is obtained by definition of the several classes of tests.

Design-Development Tests. Design-development tests, as the name implies, have the basic purpose of aiding in the development of the final design of the equipment. Since they are not usually specified in the contract or hardware specification, the flexibility in the selection of a test is quite wide compared to other types of tests. The test object may be a very early brassboard of the equipment or a scaled or full-size model of a proposed structural design, with dummy mass loading. In this case, the purpose is to obtain an early gross indication of adequate design approach together with engineering data which can be used to refine the design. This can often be best served by the selection of simple, economical test methods which may bear little resemblance to the design and test requirements but which do provide the required engineering data. For example, tests to confirm or refine the frequencies and mode shapes of the structure obtained from structural analysis would be considered in this classification. Later design-development tests may be carried out on equipment which is representative of the final design. The purpose here can very likely be described as a dry run of the qualification tests in order to detect and correct design weaknesses prior to qualification. Again, selection of test methods is still quite flexible, but, if the purpose is that of a dry run, the test conditions necessarily must be closely related to the later qualification tests.

Evaluation Tests. The term *evaluation tests* per se may not be familiar to the reader. As will be seen, there should be a distinction between evaluation tests and qualification tests, the term by which the former tests are often known. The purpose of evaluation tests is to evaluate formally the adequacy of developmental hardware as soon as available, and to identify design weaknesses or inadequacies. Usually, however, an evaluation test program does not include those tests necessary to develop and verify corrective actions taken to remove the inadequacies. The test methods and conditions employed for evaluation tests are more closely governed by contractual and specification requirements than are design development tests. However, it is usually possible to modify the methods for investigative purposes based on the results of initial tests.

Qualification Tests. The term *qualification test* has a number of synonyms, depending on both personal choice and phase of the overall program. When performed using developmental hardware, the alternative terms *type-approval test* and *proof-of-design test* are commonly used. When performed on pilot production or early production hardware, the alternative terms *preproduction* and *verification* tests may be employed. Regardless of the name, the purpose of qualification tests is to demonstrate formally the adequacy of the design for the intended use. By implication, any inadequacies revealed by testing must be remedied and the adequacy of the corrective action demonstrated as part of the qualification test program.

Those programs where significant production quantities are involved usually include periodic or sampling qualification tests for which the term *verification tests* is used. Verification tests have the same basic purpose as qualification tests and are conducted to demonstrate that neither design modifications nor changes

in manufacturing methods have introduced equipment inadequacies. These tests are usually less comprehensive than the original qualification tests, concentrating on the most severe types of test environments. Vibration tests usually are among the first selected for verification tests. The selection of test methods and conditions for qualification tests is clearly quite restricted by contractual and specification requirements.

Quality Assurance Tests. As in the previous paragraph, there are numerous synonyms for quality assurance tests, such as flight acceptance, proof-of-workmanship, delivery tests, etc. Again, regardless of the label, the common, basic purpose is to conduct a vibration test which will reveal weaknesses or defects in the equipment due to errors or excessive variability in the manufacture of the equipment. Such tests are not intended to detect design weaknesses or to demonstrate design adequacy. Unfortunately, experience indicates that quality assurance tests are too often used for such inappropriate purposes. An implicit purpose of these tests, which makes the selection of the test method quite difficult, is to accomplish the basic purpose without introducing failures or weaknesses into the equipment due to the test. In view of the intended purposes of the test (and those which are not intended), it is clear that the selection of appropriate vibration test methods for quality assurance tests is quite wide, far from unique, somewhat arbitrary, and must be based more on experience than any other test type.

Before we leave this discussion of basic test purposes, it should be evident to the reader that a normal progression of tests of the several types detailed above is likely to be applied to the successive models or versions of a particular piece of equipment. While the selection of an appropriate test method would ideally be made at each step in the progression, it is clear that precedents set during, say, design development test selection will very likely unduly restrict the later selection of qualification and even quality assurance test parameters, even though the purposes of the tests are quite distinct.

Test Object Characteristics

It is axiomatic that selection of an appropriate vibration test should take into consideration some of the characteristics of the test object. Among those which would most significantly influence the selection are value (monetary or intrinsic), size, assembly level, complexity, typicality of configuration, function, and any potentially hazardous conditions. Quantitative consideration of these characteristics is seldom possible. The judgment factors which enter into the selection are discussed generally below.

Value. The value of the test object, which may well be distinct from the value of the test, should be considered in a selection of the vibration test method, particularly with regard to procedural aspects. The term *value* was selected here, as opposed to *cost*, since a relatively inexpensive but unique test object may have a value many times its cost when factors such as schedule,

reputation, etc., are evaluated. The effort expended to select the test should, in some approximate way, be proportional to the value of the test object. First, the effort expended to define an appropriate set of test conditions should reflect the value of the test object in the sense that it is worth developing more precise and probably more complex test conditions which, in turn, will require more complex, time-consuming, and costly test procedures.

Second, the effort expended to define the test procedures should correlate to the test object value since the amount of instrumentation, the test documentation, and the measures taken to prevent test error should all reflect this characteristic of the test object.

Size. It is obvious that the size of the test object should be an important consideration in test selection. This is true from the point of view of physical size alone, regardless of weight, and also size in the sense that, within some limits, size and weight are generally proportional. In the first case, the physical size must be considered in the selection of vibration control locations and methods, the number of excitation points, e.g., is a multishaker test required, and the precision with which test conditions are known. In the second case, the additional factor of the required force rating of the vibration equipment must be determined.

Although difficult to substantiate, a general rule seems to be that "the larger the test object, the poorer the vibration test." This rule applies mainly in the context of what might be considered standard test methods applied over conventional frequency ranges and to the quality of the vibration test itself without consideration of several other test object characteristics discussed later. The rule can be defeated or at least mitigated by a selection of nonstandard test methods. The basis of the rule is found in the following factors:

1. A single vibration spectrum specified at a single point or at most a few points must become less meaningful for larger test objects.

2. The larger the test object, the greater will be the effects of impedance characteristics of the test object which are not accounted for by standard test methods.

3. The larger the test object, the greater will be the unavoidable deviations from desired test conditions due to the impedance characteristics of both the test object and the shaker/fixture combination.

4. The larger the test object of a given weight, the larger will be the required shaker force rating, due to the dissipation of vibration within the fixture, particularly in the higher frequency ranges.

5. The larger the test object, the greater will be the dissipation of high-frequency vibration with distance from the excitation points, thus increasing the risk of an inadequate test.

In the field of environmental testing, test methods for aerospace equipment were developed initially for single units, i.e., black boxes, partly because equipment was procured mainly one unit at a time and partly because available

vibration equipment could handle no more. With the trend to procurement of systems or subsystems and the availability of larger vibration equipment, a continuing trend toward the testing of larger and larger test objects has been evident. However, the original environmental standard test methods, the requirements of which are couched in terms of "the equipment shall. . .", have often been applied, or rather misapplied, to larger and larger test items, such as complete spacecraft and complete external aircraft stores. References 1 through 16 describe studies and experimental programs directed toward the development of improvements in test methods required to solve problems engendered by the size and, indirectly, the weight of test objects.

Assembly Level. At first glance, the assembly level of the test object might appear indistinguishable from test object size. However, in addition to size, the assembly level of the test object will be a significant consideration in test selection since such factors as the functional characteristics of the equipment and the possible variety of intended usages must be weighed. Generally speaking, the higher assembly levels will have more complex functional performance requirements. Confirmation of adequate performance during vibration exposure is thus more difficult and time consuming. In turn, this requires selection of a test method which allows sufficient time to measure required performance. For example, the time required to determine if a relay will chatter under vibration is far different than that required to measure the performance of a radar system.

Experience indicates that a corollary to the rule cited in the previous section is that the higher the assembly level (and therefore probably the larger the test object), the more meaningful will be the evaluation of functional performance under vibration. Some of the factors which contribute to any validity of this rule are

1. Proper measurement of the cumulative degradation of functional performance of a higher assembly level due to the incremental degradations within its component parts.

2. The difficulty of specifying the amount of performance variation or degradation in a component part which will be acceptable when the component is integrated into a higher assembly level.

3. The greater accuracy or reality with which the expected usage vibration conditions can be specified for higher assembly levels, even though it may be more difficult to test to these conditions.

A generally accepted nomenclature to describe vibration tests of various assembly levels has evolved, although there are naturally test objects which fall in a gray area between the several levels. Starting at the lowest level, these are component tests, unit tests, subsystem tests, and system tests. The selection of test methods for these various levels is discussed below.

Component Tests. Component tests are tests conducted on individual piece parts or small subassemblies such as an electronic module or a printed circuit board. Generally it is possible to test more than one sample, and frequently a

large sample size is possible. In fact, a single test may actually include as many as twenty samples tested simultaneously. Usually, the tests are performed to demonstrate adequacy of the components for a wide variety of uses and, therefore, environments.

Thus in selecting test methods for component tests, it is desirable and possible to select standardized test conditions and procedures which can be accomplished rapidly and economically on a wide range of test facilities, particularly for qualification of "off-the-shelf" items. Generally the test conditions can be quite conservative in order to envelope a wide variety of usage conditions which are typically poorly defined. For example, it is difficult enough to define the environment for a single unit or black box, let alone to define that for a single-piece part which may be used at many locations within each of the units which comprise the system. Even if it were technically possible, it would clearly be uneconomical to do so in view of (a) the typically low value and cost of the components, (b) the ease and economy with which corrective action can usually be achieved even though significant conservatism is included in the test, and (c) the most important factor that the adequacy of the components is usually more dependent on the method of packaging in the next assembly level than the configuration of the component itself. When the component for which a test method is to be defined is more specialized than indicated above, a standard test method is usually satisfactory. However, if the results of such a test indicate the need for significant design changes, the test method selected initially should be reviewed to determine if a more realistic test can be derived. For instance, as an example of the gray area between component and unit assembly levels, expensive components such as gyroscopes and display tubes, which are assembled into units, are often developed for a unique application. In such cases, the value of both the test item and the results of the test is sufficient to justify, technically and economically, the selection of more specialized test conditions and procedures.

Unit Tests. The selection of test methods for testing of single units or black boxes is influenced in part by the nature of the functional characteristics of the unit and in part by the nature of the next assembly level, if any, and associated tests at that level. Some units may constitute a complete functional system or subsystem, such as a communication set, to be installed in a carrier vehicle. In this case, evaluation of functional performance is clearcut and no higher assembly level tests will exist. In other words, the selected test is the final demonstration of adequacy by test prior to final use. In many, and probably the vast majority, of cases the unit under test is one of a number of units which together constitute a functioning subsystem or system. In this case, evaluation of functional performance during vibration exposure will be less definitive, as stated earlier, because of the absence of system interactions between the various units. In addition, the necessity of evaluating the variation of fundamental system parameters indirectly from the variation of one or more measurable parameters associated with the single unit may present a very difficult task. Selection of the

test method will depend, in part, on whether the single unit is installed individually in a carrier vehicle or whether it, combined with several other units, will be installed together in a carrier vehicle, e.g., as a complete spacecraft, and subjected to a vibration test at this higher assembly level. In the former situation, the selection of a test method is not unlike that for a single independently functioning unit. In the latter case, knowledge of the tests that will be selected for performance at higher assembly levels should influence the selection of unit test methods to ensure the compatibility of the tests at each level and to permit more liberal interpretation of functional performance degradation of the unit during test since confirmation at higher assembly level tests will be obtained later.

The appropriate test conditions for unit tests also tend to be affected by the two classes of units discussed above. A single independently functioning unit will probably be intended for a variety of uses, e.g., a radio set to be installed in a number of different aircraft. In this case, as with component tests, the test conditions may represent, by introduction of some conservatism, an envelope of expected service environments. On the other hand, a single unit falling in the second class will probably be a "tailor made" design for a single application and environment. The test conditions can then be selected to reflect the more specialized application and, to a reasonable extent, the characteristics of the higher assembly level for which the fundamental test conditions are probably defined.

Subsystem/System Tests. The classification of certain equipment as a subsystem or a system is often ambiguous. A fire control "system" may be considered a subsystem of a weapon system, for example. For purposes of vibration test selection however, the exactitude of the name is less important than two characteristics of the array of equipment to be tested. First is the characteristic that the equipment performs one or more fundamental functions which can be measured during test and which are basic to satisfactory end use; for example, to search for, acquire, and track a target of specified character. Second is the physical characteristic that, for there to be any distinction from a unit test, the equipment consists of a collection of units, most or all of which are integrated into a common supporting structure. This characteristic generally means that the test object is of more than average size and weight and that the structural aspects of the test become more significant. Further, the effects of the impedance characteristics of the test object and the structure of the service installation must, if at all possible, be taken into account in the selection of the test method and conditions.

A fundamental option in the selection of a system test method is whether the complete system is to be subjected to the vibration excitation or whether individual units of the system will be subjected to the vibration while functional performance of the complete system is observed. A third choice consisting of a mixture of the first two is obvious. When individual units, in turn, are exposed

to vibration while functioning within the system, functional performance evaluation is clearly more realistic than during a unit test. However, since only part of the system is exposed to vibration, the additive effects of degradation within several units are difficult to assess. On the other hand, it is easy to identify the culprits when degradation of performance occurs.

Configuration. It has been suggested that nothing is constant except the occurrence of changes. Since the passage of time between the selection of a test method and the execution of the selected test may certainly involve weeks and often many months, the selection process must recognize the potential though unknown changes which can (or will) occur. The most probable changes will be in the area of the configuration of the test object, particularly during research and development programs. By the time a test method has been selected, the test object manufactured, the test conducted, and the results evaluated, it will often be found that a number of design changes will have occurred which were not included in the configuration of the test object. While the effects of configuration changes will primarily affect the evaluation of test results, the test method should be selected with a view toward minimizing these effects.

Function. The function performed by the test object in its service environment must be a consideration in the selection of an appropriate test method.

First, if adequacy is to be demonstrated, the test method must be one which permits assessment of the manner in which the equipment has performed its function, either directly or by indirect means. For example, the function of a shipping container is to prevent damage to the encased equipment. Ideally, adequacy of the container is demonstrated by observing the absence of damage to this equipment after test. Frequently, however, the adequacy must be demonstrated indirectly by showing that the container does not permit vibration in excess of some level to be experienced by the encased equipment.

Second, the importance or criticality of the function of the test object must be evaluated in selecting the appropriate test method. For example, equipment whose function must be performed very precisely at a given time may require a more complex test method than equipment whose function is more general and essentially independent of time and any associated equipment, e.g., the output of an unregulated power supply.

In addition, the number of times that this function has to be performed must be included since the required duration or repetition of the test must be defined. For example, if some part of the test object must be replaced after 50 hours of operation for reasons other than vibration exposure, the test method selected to demonstrate equipment adequacy during an operational lifetime of hundreds of hours must take cognizance of this 50-hour limitation. A more obvious example, of course, is the function performed by any type of "one-shot" device, from squib-operated relays to missiles.

Magnetic Susceptibility. It is often necessary during the selection of vibration test methods to evaluate the susceptibility of the test object to environments to which it will be exposed as an incidental part of the test method. The most

common of these is the magnetic field which exists, in varying strengths for various shakers, in the vicinity of the shaker armature. The functional performance of equipment in either an absolute sense or when combined with vibratory motion may be adversely affected when the equipment is exposed to a strong magnetic field. In addition, even material properties such as damping capacity may change in some cases. While test conditions should not change for equipment susceptible to magnetic fields, the test setup and procedure may require modification to work around this susceptibility.

Hazardous Operation. The final test object characteristic to be mentioned is consideration of any factors which could contribute to creating hazardous conditions. This factor must be considered with respect to both normal operation and abnormal conditions which might occur due to failure during or at the conclusion of test. For example, the autoignition of explosive material has been experienced due to the temperature rise created by energy dissipation during exposure to vibration.

Success or Failure Criteria

Again, according to the basic purpose of vibration tests, the suitability of the test object can only be determined if some measure of suitability has been defined prior to test. Criteria for success or failure of the test and/or the test object can then be derived from this measure of suitability. In many cases, these criteria are self-evident and perhaps even trivial. For example, if the purpose of the test is merely to determine the natural frequencies and modes of a structure, there are no criteria for the test object and the test is successful if the selected method provides this information. However, if it is required that a suitable structure must have no natural frequencies in certain frequency ranges or that a minimum damping factor for each mode is necessary, then these criteria should be established beforehand and considered in the selection of the test method.

In many equipment specifications, the vibration requirements state, in effect, that the equipment "shall be undamaged by and shall provide satisfactory functional performance during and after exposure to the following vibration conditions." Of course, depending on test purpose, performance during exposure may or may not be required. Nevertheless, it is clear that criteria to define damage and satisfactory performance are needed.

If cumulative fatigue damage theory has any merit, it is probably never possible to state that equipment is "undamaged" after vibration exposure. However, it is possible to establish criteria for success based on limiting the damage. For example, the amount of wear in a bearing or other mechanical connection, the change of transmissibility or static deflection of a vibration isolator or the change in drift rate of a gyro are parameters that can be measured and used as criteria besides the obvious ones of lack of complete fracture or fatigue cracks in the test object.

Compared to damage criteria, establishment of criteria for satisfactory functional performance is usually very difficult and quite complex, depending on the

assembly level under test. As is the case with damage criteria, most test objects will exhibit some degradation, or at least change, of functional performance when exposed to vibration. Unless damage has occurred, this will disappear when the vibration excitation is discontinued. Therefore, criteria for permissible changes in functional performance must be established. It is impossible to define all these criteria in this monograph, but the following fundamental factors must be considered:

1. Is a particular mode of operation required under the vibration conditions to be simulated?

2. Is the permissible variation related quantitatively to the intended use rather than to specification values which typically reflect manufacturing variability?

3. Has the permissible variation taken into account any test acceleration factors used to establish the vibration conditions?

4. Are the spectral (i.e., frequency) characteristics of the permissible variation adequately defined?

5. Has apparent variability due to measurement error been accounted for?

6. Have permissible and nonpermissible adjustments been identified?

The next step after defining criteria for success or failure of the entire test is the definition, again before initiating test, of criteria needed for decision making when a failure has occurred or, in the case of reliability testing, when a success has occurred. Primarily, these criteria are needed during formal qualification tests and are concerned with questions such as

1. Should the test object be repaired or replaced after failure occurrence?

2. Should the test sequence be repeated or continued from the point of failure?

3. Can additional testing of an investigative or trouble-shooting nature be initiated and, if so, to what extent and of what type?

4. Should the failure be confirmed on a second test object?

5. How many, if any, failures are permissible before the test is considered unsuccessful?

6. Is the failure of such magnitude that testing should be discontinued?

By now the reader must have become aware that the definition of success or failure criteria is very closely related to the definition of test purpose discussed in an earlier section. Hopefully, he is also aware of the importance and value to be gained when these considerations are made *prior* to the *performance* of the tests. It is all too easy to fall into the trap of proceeding on a basis, best described colloquially, of, "Let's run a test, see what happens, and then play it by ear." *Tests performed with ill-defined purposes are unlikely to yield useful and valid results.*

It may not be so obvious that consideration of success or failure criteria *prior* to *selection* of test method is almost as important. The method selected may

well be very different, depending on whether these criteria are of a "go- no go" type or of a threshold type. For example, a different test method would be selected depending on whether it is required that a relay not chatter under certain conditions or whether the level at which chatter will occur, as a function of frequency, is to be measured. As another example, when the complete vibration test consists of several parts, each with differing detailed purposes, the selection of test method should reflect this situation. For example, when part of the test is to demonstrate functional performance and part is to demonstrate structural integrity, it is often advisable to complete all testing for the first part prior to starting testing for the second part. Again, if a test to failure under swept sinusoidal vibration is to be conducted, a large number of relatively short-duration sweeps is preferable to a small number of very slow sweeps since the former will provide better resolution of time to failure.

It is not intended to suggest that the criteria discussed above will all be found entirely satisfactory once testing has been initiated. After all, if one knew everything that would happen, it would probably be unnecessary to run the test. However, if a set of criteria have been established in the relative calm prior to test, any modification or additions to the criteria which develop and the necessary engineering decisions required under the usual pressure of test conduct will be established on a more rational basis.

Replication of Tests

Because of the inherently destructive nature of vibration tests compared to many other environmental tests, the opportunities to replicate vibration tests are rather rare. Yet experience and the literature illustrate the wide variation of fatigue and damping properties of almost all materials and thus indicate the desirability of replicating vibration tests, i.e., conducting the "same" test on a number of "identical" samples, using the principles of statistical design of experiment to yield significant test results. The opportunities to replicate tests generally are found in component or piece-part testing and in quality assurance testing of higher assembly levels.

Significance of Test Results

The results of vibration tests may have significance in a number of nontechnical areas, such as cost, schedule, etc. The consideration here, however, is the technical significance or validity or value of the test results. Most often, this consideration arises in connection with failures during test but probably should be made more frequently in connection with successful tests. If this consideration is made after testing is initiated, clearly no effect on selection of test method will occur. However, prior consideration of this factor will often assist in

selection of the optimum method. Typical of the questions which must be answered to evaluate the significance of the test results are

1. Is the configuration of the test object representative of the equipment in use?
2. Are the vibration test conditions well defined and an adequate simulation of service conditions?
3. Are the excitation and control methods satisfactory?
4. Is the test object attachment realistic?
5. Is the test sample size adequate?
6. What is the significance of variability in both test object and test conditions?

2.2 Test Conditions

The general considerations in the selection of a vibration test discussed in the previous sections have been primarily indirect factors in the selection process. In this section, the parameters which must be selected to properly define the test conditions are discussed. These parameters include descriptions of the vibration itself with recognition of the inherent simulation characteristics, the locations for excitation of the test object and control of the test level, the required data, and finally, the required accuracy.

Selection of Vibration Conditions

The vibration conditions which must be specified to define a vibration test are (1) excitation parameter, e.g. motion, force, etc.; (2) waveform, (directly or indirectly); (3) frequency range; (4) duration; and (5) level as a function of frequency. Each of these conditions can generally be specified independently, although the simulation characteristics of the test are affected by interrelationships between these parameters.

The actual selection of the conditions can be made through three basic approaches. First and most directly, when the purpose of the test permits, is the selection of a set of conditions which reproduce the expected environment to the greatest extent that is technically and economically feasible. The second approach is less direct and is based on selection of a set of test conditions which will cause the same effects to be manifested in the test object as would exposure to the expected environment. The third approach is essentially an arbitrary selection of conditions, based mainly on precedent and experience, which will achieve a specific purpose without regard to the simulation considerations inherent in the first two approaches.

Excitation Parameter(s). When the vibration conditions are to be described in terms of a single excitation parameter, the selection is rather simple since motion (i.e., displacement, velocity, acceleration) and applied force are the two parameters which can be created and controlled in the laboratory. When the description of test conditions is in terms of both motion and applied force, some form

of impedance testing is prescribed. Since the excitation at a particular point produced by a single shaker can only react to one control signal at any instant, such a description may take two basic forms. In one case, the motion is specified together with the constraint that some limiting force is not exceeded, or vice versa. Thus the excitation parameter may shift from motion to force and back to motion at various points in the frequency range. In the second case, the excitation parameter is defined in terms of some mathematical combination of motion and force, usually as a function of frequency, which is an impedance test in the truest sense.

Clearly the selection of the excitation parameter depends on the data available to define the other test conditions discussed below. Since measurement of the dynamic service environments is almost entirely in terms of motion, it is not surprising that almost all vibration tests are specified in terms of motion.

Waveform. The selection of the appropriate waveform which defines the time variation of the excitation parameter is essentially limited to five practical possibilities: (1) sinusoidal, either fixed or variable frequency; (2) random with approximately Gaussian statistical properties; (3) a combination of 1 and 2; (4) complex periodic waveforms; and (5) playback of recorded time histories. The circumstances in which the selection of a particular waveform may be made are described qualitatively below.

Simulation of Environment. If the basic approach to the selection of test conditions is to be followed, i.e., reproduction of the service environment, then the selected waveform should reproduce the essential deterministic and/or statistical characteristics of that environment. It might appear that the fifth option listed above would be the immediate choice in this case. However, for a number of reasons, such as the atypicality of the recorded time history, the unknown and uncontrollable effects of amplitude and phase distortion, the inherent risk of open-loop test control, etc., it is believed that the apparent advantages of this method of achieving the desired waveform are largely illusory and that this approach is suitable only in very special and restricted circumstances. If the time history is not to be reproduced, then the major waveform characteristics which must be reproduced are (1) the spectral characteristics, i.e., the variation of intensity with frequency; (2) the statistical characteristics of either the instantaneous or peak values of the waveform in terms of the appropriate probability density functions or correlation functions; and (3), when multiple control-excitation parameters are involved, the interrelationships between these parameters, such as relative phase, co- and quadspectral densities, or cross-correlation functions. While a discussion of the determination of these waveform parameters from service environment data is beyond the scope of this work, it can be said that fixed- or variable-frequency sinusoidal waveforms rarely reproduce the desired characteristics of the service environment and thus will be infrequently selected if the environment is to be simulated. On the other hand, it has been found that the waveform characteristics of a random noise signal with Gaussian

or normally distributed amplitudes and appropriate spectral shaping will generally reproduce the essential characteristics of the service environment and lead to selection of the second option above. Exceptions to the above statements are found in several instances. For example, the vibration due to high-rate gunfire in aircraft may be readily simulated by a pulsed or complex periodic waveform as described in Chapter 3 [17]. Again, the vibration in helicopters includes motion at the fundamental rotor frequency and its harmonics, i.e., complex periodic motion, which, conceptually at least, could be selected as the waveform for testing purposes. The selection of a combination of sinsoidal plus random waveforms achieved some popularity for testing of early spacecraft when it was found that certain solid rocket motors exhibited a "screech" which could be characterized as a sweeping sinusoid superimposed on the typical broadband random excitation. Clearly this basic approach to the selection of waveform can only be followed if sufficient data to describe or accurately predict the usage environment are available. If such data are available, then selecting the appropriate waveform is quite straightforward.

Simulation of Environmental Effects. The approach of selecting the vibration waveform for test which will simulate the effects of the vibration waveform encountered in service has several implicit limitations. First, it implies that the effects of the service environment on the probably as yet unused equipment are known. Second, it implies that the effects of the test environment on the as yet untested equipment are also known. Third, regarding waveform, it is implied that the relationships between the service waveform and service effects and between the test waveform and test effects are understood. A little reflection is sufficient to come to the realization that this approach can be taken only on the basis of past experience with similar equipment and as a means of modifying the previous approach of direct simulation of the environment to achieve more practical and economical test conditions.

Arbitrary Selection. Selection of waveform based on either simulation of the environment or simulation of the effects of the environment is implicitly related to tests intended to demonstrate adequacy in a service environment. As discussed previously, a number of tests have purposes which are only indirectly related to the service environment and for which, therefore, the waveform may be selected arbitrarily to best suit the purpose of the test.

The most common example of this situation is found in those tests conducted to determine the dynamic characteristics of the test item, i.e., the natural frequencies and modes, the frequency response or transfer functions, etc. Two basic waveforms may be selected for this purpose. First, a slowly swept sinusoidal excitation over the desired frequency range may be used. Alternatively, a broadband random excitation, typically but not necessarily with constant spectral density, may be used. A third possibility of using a shock or impulsive excitation is described in the literature [18,19]. The choice between the swept sinusoidal and broadband random excitations should be based on the following factors:

1. The nature of the waveform specified for the design requirements.

2. The nature of the waveform expected in the service environment.

3. The availability of tracking and other ancillary equipment for the analysis of swept sinusoidal signals.

4. The availability of spectral analysis equipment for analysis of random vibration signals.

5. The duration of excitation required.

6. The most convenient format for further processing of the analyzed data.

7. The relative total cost of the two approaches.

Following chapters describe the above factors in greater detail. It is appropriate, however, to indicate here, in qualitative terms, the manner in which these factors affect the choice of approach. If the test item were a perfectly linear system, the first two factors would be immaterial. However, recognizing the inherent nonlinearities of physical systems, it is desirable to measure the transfer functions, etc., with the same excitation waveform (and intensity) as service excitations in order to minimize the complex effects of nonlinearities, particularly those evident in the damping properties.

The next two factors are obviously interrelated. Again due to nonlinearities, this time in both test equipment and test item, it is found that the sinusoidal excitation and response will be rich in distortion at many frequencies, particularly at the natural frequencies which are generally of most interest. Therefore, to obtain valid transfer functions, it is necessary to remove this distortion from each signal by filtering prior to the comparison of the relative amplitudes and, when necessary, phase of the two signals. When random excitation is employed, rapid and accurate spectral analysis equipment must be available. If single-point excitation is employed, simple power (or auto) spectral density analysis will suffice, providing phase relationships are not required. If multipoint excitation and/or phase relationships are required, it is necessary to compute cross spectral densities (both co and quad) in addition, thus complicating the data reduction requirements. An alternative and analagous approach to data analysis of random excitation is the use of auto- and crosscorrelation analyses, combined with Fourier transformations. In terms of data reduction complexity and equipment requirements, the spectral and correlation methods are approximately equivalent.

The fifth factor relates to the possibility of damage to the test item during exposure to the selected excitation. Since the complete excitation frequency spectrum is excited by broadband random excitation, this approach permits the required data to be obtained during a very short exposure, of the order of 10 to 20 sec. It should be noted that the usually stringent requirements on the bandwidth-time product for statistical accuracy of spectral analyses do not apply since the statistical errors effectively "cancel out" when the spectra are ratioed to obtain transfer functions, provided the same time sample of data is used for each signal.

The sixth factor regarding the further processing of the analyzed data should probably receive the most consideration but often is given scant attention. With

the increasing use of digital computation in structural and dynamic analysis, the outputting of reduced data from tests in digital form for further processing, such as comparison of experimental and analytical results, becomes increasingly desirable from a time and cost standpoint. At the present time, random vibration data are more readily adaptable to such formatting than those obtained from sinusoidal excitation. In any case, proper design of the complete experiment requires that the complete data analysis and evaluation process be considered when selecting the type of excitation for the test phase.

Regarding the last factor, it is again necessary to examine the complete experiment to determine the relative costs of the two approaches. A relatively inexpensive test which yields data which must be laboriously transcribed for evaluation may well be a poor bargain compared to a more expensive test which yields the required data in a convenient format.

A further example of the arbitrary selection of vibration waveforms arises in the selection of waveform for quality assurance or proof-of-workmanship tests. The objective here is to select a waveform which will efficiently reveal defects in the test item while avoiding the occasion of damage. All the traditional waveforms have been employed for this purpose. In addition [20,21] complex periodic waveforms with rich harmonic content, such as those produced by certain reaction-type vibrators with impact loading, have been used widely. The adequacy of the waveform selected can only be judged after the fact, based on the subsequent failure history in equipment so tested. However, it has been observed that those waveforms which may be considered broadband, whether deterministic or not, do appear to be relatively more efficient in revealing workmanship errors in assembled equipment. For example, detection of insufficiently torqued screws, missing lockwashers, etc., is quickly achieved.

Frequency Range. Selection of the frequency range over which the vibration intensity is to be specified for a test usually requires little more than some common sense. First, the frequency range over which the available vibration test equipment can provide the required vibration intensity and waveform provides lower and upper bounds. Displacement capability typically limits the lower frequency cutoff, while the frequency response of the complete vibration system, including control equipment, defines the upper frequency limits. Beyond equipment limitations the purpose of the test in conjunction with the dynamic characteristics of the test item, either known or estimated, can be used to limit the required frequency range to avoid unnecessary expense in both testing and data processing. Since response data of interest and equipment damage are generally observed at the natural frequencies of the test item, testing more than an octave below the lowest natural frequency of the test item is unlikely to be particularly fruitful (unless one is calibrating a transducer, for example). On the other hand, there is little to be gained by testing to an upper frequency which is beyond the upper frequency at which damage or malfunction occurs, or beyond which either the excitation intensity or response characteristics are known or understood. In fact, experience indicates that specification of

test conditions to unnecessarily high frequencies frequently leads to serious misinterpretations. For example, with sufficient bandwidth, rather large overall rms accelerations result from moderate spectral density values. While this example may seem almost pathetic, it is nevertheless real.

Test Duration. The selection of test duration discussed below and the selection of test level discussed in the next section are very closely interrelated. With few exceptions physical failure during vibration occurs through fatigue of materials. Thus the likelihood of failure is directly coupled to the duration of the test.

The motto of a vibration test activity might well be, "If we shake it hard enough or long enough, we can break it." However, if the duration is to be selected by one who is less destructively inclined, then three options are available: (1) duration based on simulation of service life, (2) duration which will uncover a satisfactory fraction of potential failures, and (3) duration which will achieve the purpose of the test.

Selection of a test duration based on operational life may be very straightforward, such as in the case of spacecraft and boosters, ground-launched missiles, etc., where simulation of the complete vibration exposure amounts to a few minutes' test duration. On the other hand, direct simulation of the vibration exposure of airborne equipment which may last for hundreds of hours over a wide range of intensities is completely impractical. In this case, a test duration must be derived which, based on some acceptable model for fatigue damage accumulation, is equivalent to the service environment. This derivation would logically lead to a test duration at the maximum expected intensity which is equivalent to the integration of the cumulative effects of varying durations at varying intensities up to and including the maximum expected intensity. This might not be considered an accelerated test in the usual sense of the term even though simulation of the service life is accomplished in an accelerated manner. If the test duration so derived is still impractically long, then the duration of an accelerated test in the usual sense of the term, conducted at an intensity greater by some factor than expected in service, may be derived using the same model for fatigue damage accumulation. It should be noted that the above derivations should be carried out independently of any factor of safety or ignorance which is to be arbitrarily applied to either duration or intensity. Chapter 3 describes quantitatively the methods of deriving test durations in this fashion. An approximate rule of thumb relating duration and intensity is that a 3-dB increase in intensity (doubling of spectral density) is equivalent to a factor of ten in reduction of duration.

As discussed further in Chapter 3, the approximate duration to be simulated may be described either in time, yielding more cycles of motion at higher frequencies, or in cycles, yielding shorter time durations at higher frequencies. The latter case is more likely under the second option mentioned previously when a certain fatigue life, in cycles, corresponding to an effective endurance limit, is to be demonstrated.

Laboratory experience in vibration testing seems to indicate that, for a given vibration intensity, most failures that are going to occur will occur in the first few minutes of test, regardless of the type of vibration waveform, etc. This experience is substantiated by the experience in product assurance testing described by Kirk [21]. In this program, consisting of some 11,000 tests, it was found that essentially all workmanship failures occurred within 15 min. If these results may be considered typical, a logical means of selecting a test duration which will detect a satisfactory percentage of potential failures is thus available. Of course, the foregoing seems to fly in the face of cumulative fatigue damage theory. However, if one postulates that most failures in vibration tests are initiated by an imperfection of some kind which causes severe stress concentration, then failure is due more to exceeding the ultimate strength or the low cycle fatigue life rather than the sloping portion of the normal endurance curve to which cumulative damage is applicable.

Although the third option listed above might be considered a catchall covering any situation in which the first and second options do not apply, logical selection of minimum durations, based on the test purpose, may be made in the following cases:

1. Selection of duration required to verify satisfactory functional performance, i.e., how long does it take to check out the equipment. This is often used in conjunction with an accelerated test during which degraded performance is allowed.

2. Selection of a duration consistent with the capabilities of vibration test equipment, e.g., a sweep rate slow enough to permit accurate level control by the servos.

3. Selection of a duration consistent with minimum data requirements for data analysis procedures. For example, a sweep rate slow enough for accurate analysis with tracking filters, X-Y plotters, etc. Or, as another example, a duration sufficient for adequate statistical accuracy in analysis of random vibration.

4. Selection of a duration consistent with achieving desired test object response. For example, a sweep rate slow enough to permit quasi steady state response (see Chapter 3) or fast enough to simulate a transient excitation.

5. Selection of a duration consistent with instrumentation and data acquisition capabilities, e.g., the length (timewise) of a reel of magnetic tape.

Many similar bases for selection of test duration in this category will undoubtedly come to the reader's mind.

The most difficult problem in the selection of test duration, particularly if it is to be based on simulation, has to do with the direction or directions of excitation. Vibration tests are typically conducted for a stated duration in each of three orthogonal axes, one axis at a time. Generally, if the environment to be simulated calls for a certain duration, say T min, then the test will consist of excitation for T min in each axis, for a total duration of 3T min. However, it is an unusual test object which does not respond rather omnidirectionally, at least

in some resonant frequency bands. Furthermore, it is unusual if the vibration exciter, when loaded, does not produce significant crosstalk excitation in a few frequency bands. This approach is thus demonstrably conservative. However, the degree of conservatism is unknown and therefore a determination of the appropriate reduction of test time per axis is impossible. However, the conservatism is probably less penalizing than that engendered by misguided attempts to solve the problem by exciting in one direction only at a level whose vector components in the three orthogonal axes are equal to the required levels; i.e., for equal components, test at an amplitude of $\sqrt{3}$ or 1.73 times the specified amplitude.

Test Level. A detailed discussion of methods of selecting vibration test levels is far beyond the scope of this monograph, particularly where the test level is intended to simulate a service environment. The first monograph in this series by R. H. Lyon (SVM-1), *Random Noise and Vibration in Space Vehicles,* [22], was addressed to this topic for space vehicles. The extensive bibliography in that work and Refs. 23 through 27 of this monograph will lead the reader to the appropriate literature regarding the prediction of vibration environments for various classes of equipment. However, the next step of translating a given measured or predicted environment into a meaningful test is probably as important and certainly equally difficult.

In fact, two steps are really involved here. First, the environment must be translated into a meaningful set of design requirements. The second step then consists of developing a set of test requirements which will demonstrate compliance with the design requirements.

In principle, differences may logically exist between the design requirements and test requirements. These differences may reflect inherent limitations of vibration testing equipment such as maximum displacement, minimum or maximum practical frequency range, output power capability, etc., in addition to such factors as test acceleration, uniaxial testing, etc. In practice, it is unfortunately true that the test requirements frequently tend to become the governing design parameter due to two major inadequacies in present methods of selecting or deriving test conditions, which lead to unknown conservatism, particularly in test levels.

First, the inability to account adequately for the differences in impedance characteristics between the usage installation and the test configuration leads to the definition of test level in terms of input motion to the test item. Such a definition automatically creates a test configuration which effectively has infinite output impedance at the interface between the test item and the test fixture. In other words, no matter how the test item responds to the excitation nor how much force is required to create the required motion, the test level is unchanged.

Second, test levels are generally based on data measured under a variety of conditions at a number of locations on a structure or equipment which is hopefully representative of that for which a test (or design) level is to be selected. In some cases, it has been possible to modify the data previously

gathered to the present structure or equipment by use of analytical or statistical techniques. In either case, the test level is generally selected by enveloping all, or almost all, e.g. 95 percent, of these data in order to define an input vibration level. While safely conservative, this approach, in effect, discards the information contained in at least 95 percent of the available data.

It is clear that these two inadequacies are interrelated and both derive in part from the following philosophical point. Any measurement of vibration, either force or motion, is actually the measurement of vibration response at a particular point of a given dynamic system to a generally unknown and probably unknowable excitation. This is true whether the measurement is made in the laboratory or in the usage environment. Recognition and acceptance of this fundamental point then make it clear that the definition of a vibration "input" is an artifice employed to define a vibration test in reasonably simple terms. While it is generally necessary to resort to this artifice, the level which is selected as an "input" should take cognizance of the fact that it is indeed a response. References 1, 28, and 29 describe a technique proposed to mitigate the inadequacies discussed above.

Although beyond the scope of this monograph to explore the selection of test levels in detail, it is appropriate to indicate the steps that should be employed in the following situations:

1. Simulative tests to which the service environment is known or has been predicted.

2. Simulative tests for which the service environment is unknown.

3. Design tests and investigative tests where there are essentially no prior constraints.

4. Quality assurance tests.

Each of these situations can be envisioned for the various equipment assembly levels, i.e., piece-parts up through complete systems, and the level selected should reflect the assembly level of the test item as well as the particular situation. The previous discussion on page 11 should provide the reader with sufficient assistance to properly allow for this factor.

Test Levels for Known Environments. If the environment may be considered to be known, it will typically be defined in either the equipment specification or a set of design requirements. This definition will often describe the vibration input, omnidirectionally or in three orthogonal directions, to the complete equipment assembly, e.g., the spacecraft. In addition, limitations on the response of the equipment at certain locations may be specified. Selection of the test level is then trivial.

If the equipment assembly contains a number of units or major subassemblies, the above definition may also include vibration test levels for these individual parts of the equipment, which again makes selection trivial. If this definition is not included, it is usually necessary to select a test level which will provide high confidence that these units will be satisfactory when installed in the

complete assembly of equipment. What would appear to be an appealing approach to a solution of this problem is the determination of the transfer or frequency response function between the specified input location and the unit attachments, either by analysis or by test of a structural model of the complete assembly. Multiplication of this frequency response curve by the specified input would presumably then yield the appropriate unit test level. However, test levels obtained in this manner must be tempered with engineering judgment. First, the test levels would undoubtedly vary with frequency in a very complex fashion, thus leading to very complicated tests. Second, the dissimilarities between the real equipment and either a laboratory test model or an analytical model would render the fine detail of the frequency response curves essentially meaningless. Furthermore, it is likely that the high end of the frequency range would be attenuated to an unrealistically low level when the real environment is considered.

The most difficult judgment to make will be with respect to the several large peaks in the derived test levels which reflect the primary modes of the entire structure. It requires a degree of courage and, of course, conviction to select a test level which does not envelop these peaks in both amplitude and frequency. Yet consideration of the probable inaccuracies in the derivation process, the unknown effects of impedance mismatch between unit and equipment assembly, the very significant differences between installation of the unit in a very rigid vibration fixture and the relatively flexible assembly, and the probable penalties due to unnecessary conservatism, requires that a test level based on smoothing or averaging of the transfer functions, rather than enveloping, be selected. The even more difficult task of selecting test levels for components (piece-parts) was discussed on page 11.

Test Levels for Unknown Environments. When it is not possible to determine or reasonably predict the service environment, selection of the vibration test level may be made in one of two ways. First, test levels which previously have proven satisfactory for similar equipment or for similar use, i.e., spaceborne, airborne, etc., may be used again. Alternatively, general Government specifications such as MIL-STD-810 should be consulted. These specifications usually contain several alternative test procedures, each of which may be conducted at one of several levels for a given duration. In addition, guidance in selection of the appropriate method and test level is included, based on the size, location in the vehicle, type of vehicle, propulsive system, type of installation, use of vibration isolators, etc. While it is generally conceded that these specifications call for quite conservative tests, the benefits which accrue from selection of a standard test should not be overlooked. These include such factors as familiarity of designers with such requirements, relative ease of conducting tests, applicability for additional applications of the equipment, etc. In the case of components or piece-parts, use of these specifications is recommended, even when the testing of the next higher assembly may consist of quite advanced test methods or nonstandard test level descriptions.

Test Levels for Design/Investigative Tests. Test levels selected for design and investigative tests, as suggested for the selection of other test condition parameters, should bear close relationship to the levels specified for later formal testing. Since the hardware available for this type of test is often intended to serve several other important purposes, it is generally prudent to select levels which are, initially at least, depressed from the formal test levels. This approach of gradually building up to full level has several rather obvious advantages. First, in case of design weakness, the opportunity to detect and understand the weakness before catastrophic failure is enhanced. Second, the threshold at which failure occurs may be ascertained, thus giving a measure of the degree of design modification required. Third, when several design deficiencies exist, they may all be identified prior to corrective action and retest, thus minimizing the number of iterative design and test cycles required to achieve a satisfactory design.

Excitation and Control Locations

The definition or specification of the locations of the vibration excitation and the test level control transducers to be used for a vibration test is often only loosely defined in vibration test specifications, even though the outcome of the test may be strongly influenced by these two parameters. Thus the vibration test engineer is in a position to select these test conditions and influence the quality of the test more frequently than for any of the parameters discussed so far.

The situation arises, of course, from the fact that the vibration specification is generally written at about the time the equipment design is initiated. It is thus impossible to describe these locations physically or dimensionally and they must therefore be described in general terms which provide guidance to the test engineer when he ultimately selects the exact physical location. Frequently, the final selection is and can only be made after the equipment is installed in the vibration fixture ready for test.

In the vast majority of tests, the excitation and control locations are identical, permitting discussion of these parameters in the same section. For certain specialized tests, additional control locations are employed, requiring some separate discussion in the following subsections. Location of response transducers used to monitor the response of the test item has little hopefully, no - effect on test performance and will not be discussed.

Input Versus Response. The discussion of test level selection emphasized the fundamental fact that the vibration measured at any point represents the response of a dynamic system to some excitation, and that it is only a necessary artifice to ascribe the characteristics of an "input" to such a response. The significance of this point in selecting excitation and control locations is illustrated in Fig. 2-1. If, as illustrated in Fig. 2-1a, the vibration excitation can be described as a single-point excitation at some point of the test object with an input impedance Z_i, then the response of that item to a given force or motion excitation F_i or V_i is unique and independent of the output impedance of the

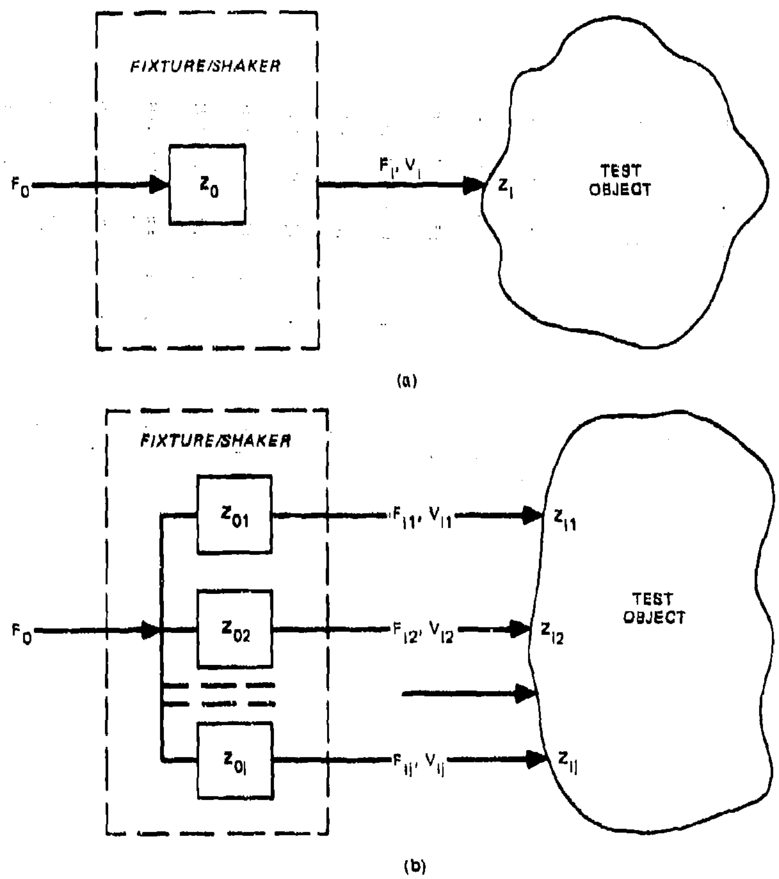

Fig. 2-1. Excitation points: (a) single and (b) multiple.

shaker/fixture system Z_0. Z_0 only defines the ability of the shaker/fixture to create the desired excitation. However, this is not the usual situation and Fig. 2-1b illustrates conceptually the more common situation where the attachment to the test object is made at a number of points. There will be differences from attachment point to attachment point of both the input impedance to the test object and the output impedance of the shaker/fixture system. Thus when the shaker is excited, the motion at the various attachment points, i.e., the responses of the system at those points, will be a function of the dynamic characteristics, i.e., impedances, of both the test object and the shaker/fixture. Thus the same test object, tested to the same levels using different shakers and/or test fixtures, will exhibit different responses. It should be clear that these differences will tend

to increase in magnitude as the excitation frequency increases. Of certainly equal and probably greater importance are the differences between output impedances of the shaker/fixture and the supporting structure in the service installation. Although it is generally impossible to adjust quantitatively the test conditions for these effects, the proper selection of attachment points and control locations as discussed in the following sections can at least mitigate these effects and frequently avoid the generation of completely unrealistic responses in the test object.

Excitation Locations. In the majority of vibration tests, the selection of excitation location is trivial, since the only feasible locations are the normal attachment points of the test item. Then a test fixture is built which will introduce, insofar as possible, the required motion at each of the attachment points, leading, of course, to the design of very rigid, heavy fixtures. The next section discusses methods of controlling the excitation which accommodate the almost inevitable differences in motion between attachment points. It is clear that, with this approach, the excitation will be introduced at the appropriate locations but that, perhaps with the exception of vibration isolated equipment, little simulation of the service installation is achieved. On the other hand, it is usually possible to achieve the desired test levels at these points. A rather radical departure in customary test specification would be required to permit use of fixtures which more nearly simulate service installation [30].

Generally, the testing of components, units, and even small subassemblies is accomplished as outlined above, in part because it is the most feasible way and in part because it is at the attachment points that the environment is defined, can best be predicted, and would normally be measured during a field measurement program. As the physical size and, to some extent, weight of the test object increases, the above approach is generally unsatisfactory. Selection of the excitation location should now focus more on the manner of excitation in the service environment rather than on only the attachment points.

For example, in the case of an external store on an aircraft (including the case of a weapon bay when the doors are opened) the excitation due to aerodynamic flow, etc., is applied to the exterior of the store and "flows" into the aircraft through the launcher and pylon. Any attempt to vibrate the store by excitation through the launcher hooks is, to carry the analogy one step further, swimming upstream and, based on experience with a number of air-to-air guided missiles, will not provide an adequate vibration test. A likely result of such an approach is the early and unjustified failure of the launcher hooks before the desired missile vibration level is achieved. Furthermore, the design of an adequate fixture and creation and control of the test level at generally well-separated points on a massive test object is difficult if not impossible. It is therefore appropriate to apply the excitation to the store at a number of convenient points, such as main structural bulkheads or the motor thrust ring. References 1 and 4 describe the results of such an approach. The main point here is that the test must be designed to create a certain response level within the test object and that the use

of an "input" approaches the ridiculous. To paraphrase Dreher [23], a 1000-pound missile cannot be tested using an environmental test procedure written for testing black boxes.

Not all massive test objects are amenable to the approach of the above discussion. A complete spacecraft or payload is typically attached to interface structure by being bolted or clamped to a ring. The excitation location can usually be only at this ring, due to the characteristics of the spacecraft structure and configuration. In this case, it is recognized that the excitation is both mechanical and acoustic. Mechanical excitation acts through the interface while acoustic excitation acts over the entire spacecraft. Thus excitation of the interface as an input is appropriate, provided some means of controlling the response of the spacecraft to appropriate levels is included.

Control Locations. It was not too long ago that the test level of a vibration test was generally "controlled" to the desired value by use of a velocity transducer mounted on the end of the shaker armature away from the test object. Fortunately some progress has been achieved since that time and the locations employed for test level control are now somewhat more meaningful. For the majority of tests conducted as described in the beginning of the previous section, the obvious location for control transducers, generally accelerometers, was on the vibration fixture adjacent to the test object attachment points. For example, MIL-STD-810B, Method 514, Paragraph 5.5 states, "The input monitoring transducer(s) shall be rigidly attached to and located on or near the attachment point or points of the test item." While it is not completely clear which side of the attachment point is intended by this statement, it is customary to mount the transducers on the fixture. This is desirable for several reasons, such as the availability of flat rigid surfaces, the avoidance of marring the test object finish, the ability to screw down rather than glue on the accelerometer for test safety, the repeatability of tests, etc.

For the control of some of the nonstandard tests discussed in Chapters 3 and 4 and briefly in the previous section, the control transducers must be located at additional points on the test object as well as at the point of excitation. Selection of these locations must be compatible with or analogous to the locations for which the test levels were derived, predicted, or measured. Generally these would be the attachment points between the units which make up the assembly under test and the major structural members of the assembly. Thus the test engineer requires a reasonable knowledge of the origin of the test levels to make a meaningful selection of control locations for these specialized test methods.

Level Control Method

As was true for control locations, the definition of level control method is often either omitted or only loosely specified in most vibration test specifications. Until about 1965, it was generally unnecessary to define the control

method since it was only infrequently that more than one transducer would be employed to provide the control signal which was then used directly as an input to the vibration test equipment. In the past five years, it has become quite common, in fact almost standard, to use multiple transducers to generate control signals which are processed in one of several ways prior to injection of a single signal into the vibration test equipment for control of the test level at any particular time, either manually or automatically by use of servoamplifiers. Because of rather fundamental differences of signal characteristics, the techniques employed for sinusoidal and random vibration are sufficiently different to warrant discussion separately. However, there are one or two fundamental reasons for electing to employ several control transducers and these are common to every test waveform.

As mentioned previously, most test specifications or procedures call for a certain vibration level to be applied at the equipment attachment points, with the implication that the motion time histories will be identical at all attachment points. Very little analysis or practical experience is needed to convince one that there will be significant differences between these motions, particularly in high frequency regions, say above 1 kHz. For larger test items, these differences may be evident as low in frequency as 100 to 300 Hz, whereas for testing of small components (piece-parts) the differences may not occur below 2 or 3 kHz. The above rules of thumb assume that a "good" vibration fixture on an adequately larger shaker is available for use. The cited frequencies can be reduced significantly with a poor fixture, an undersized shaker, or a particularly bulky test item. In any case, differences will usually exist within the desired test frequency range and be greatest at frequencies where one or more of the attachment points become either nodes or maximum response points of the shaker-armature/fixture/equipment dynamic system. When a single control transducer is employed, severe overtest or undertest, respectively, will occur, even if the vibration test equipment is able to cope with the required changes in driving signal. To accept that this situation does constitute overtest or undertest is perhaps a matter of philosophy. It does seem reasonable that the specified test level is unlikely to have been derived for a single, arbitrarily chosen attachment point of the equipment under test and is more likely derived as a number representative of the motion of all or any of the attachment points.

A second reason for employing multiple control transducers is concerned with the empirical simulation of impedance effects. Although related to the first, the second reason is somewhat different. In the first case, the use of multiple transducers is a recognition of the impossibility of creating the desired identical motion of the several attachment points. In the second case, the use of multiple transducers is a recognition that it is often desirable to permit, or even encourage, differences at the various attachment points to obtain an improved simulation. The "loading down" of the attachment points due to the impedance of the test object is allowed to occur as naturally as possible.

To illustrate the foregoing, consider the vibration fixture shown in Fig. 2-2. This fixture, which weighs 173 lb, is used to test avionics units weighing 30 to 50 lb on a 30,000-lb exciter. It is quite rigid and supports the units through four pins and bushings, two at each end of the unit, and a screw-type latch at the front. During random vibration testing in the vertical direction, four accelerometers were placed as shown in Fig. 2-2 for test level control. The variation of acceleration spectral density among these four locations during excitation at 0.1 g^2/Hz from 20 to 2000 Hz is shown for two units, one weighing approximately 30 lb and the other approximately 40 lb, in Figs. 2-3 and 2-4, respectively. In these figures, the two curves represent the maximum and minimum spectral densities of the individual accelerometer signals, divided by the average of the four individual spectral densities, in each 10-percent analysis bandwidth. If there were no variation, the curves would be coincident at a value of unity. It should be noted that these curves envelop the four individual spectra and do not represent the spectrum at any individual location. While generally similar, the differences between the two units are apparent. It is clear that very different tests would be conducted if any individual accelerometer were selected for control. In addition, since the test level is representative of a zone of an aircraft fuselage and thus the average of the motion at individual mounting points, the expected environment is better simulated by controlling the average of these four accelerometer signals.

Sinusoidal Test Level Control. Figure 2-5 illustrates the several choices to be made in defining the test level control method for sinusoidal tests for either single or multiple control transducers. It will be seen that certain of the paths will also be applicable for other nonrandom waveforms. The first decision point

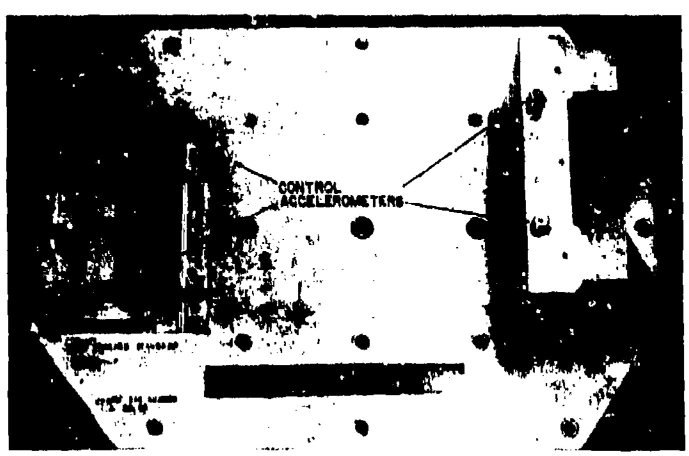

Fig. 2-2. Avionics unit fixture.

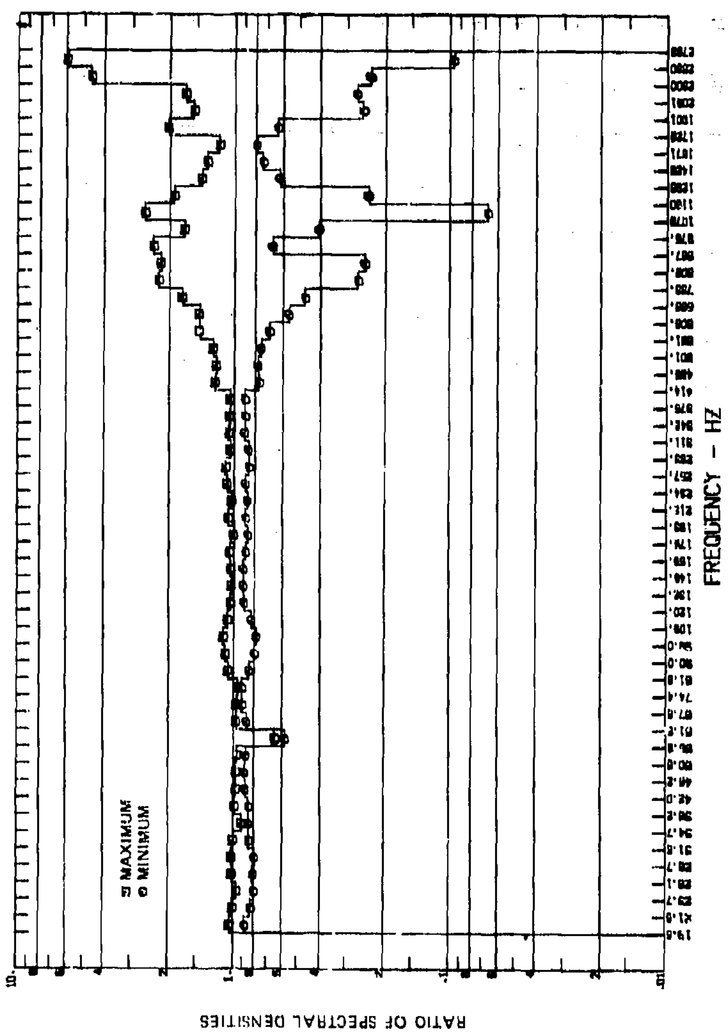

Fig. 2-3. Maximum and minimum variations of individual control accelerometer spectral density from power average of four accelerometers on fixture shown in Fig. 2.2 loaded with 30-lb unit.

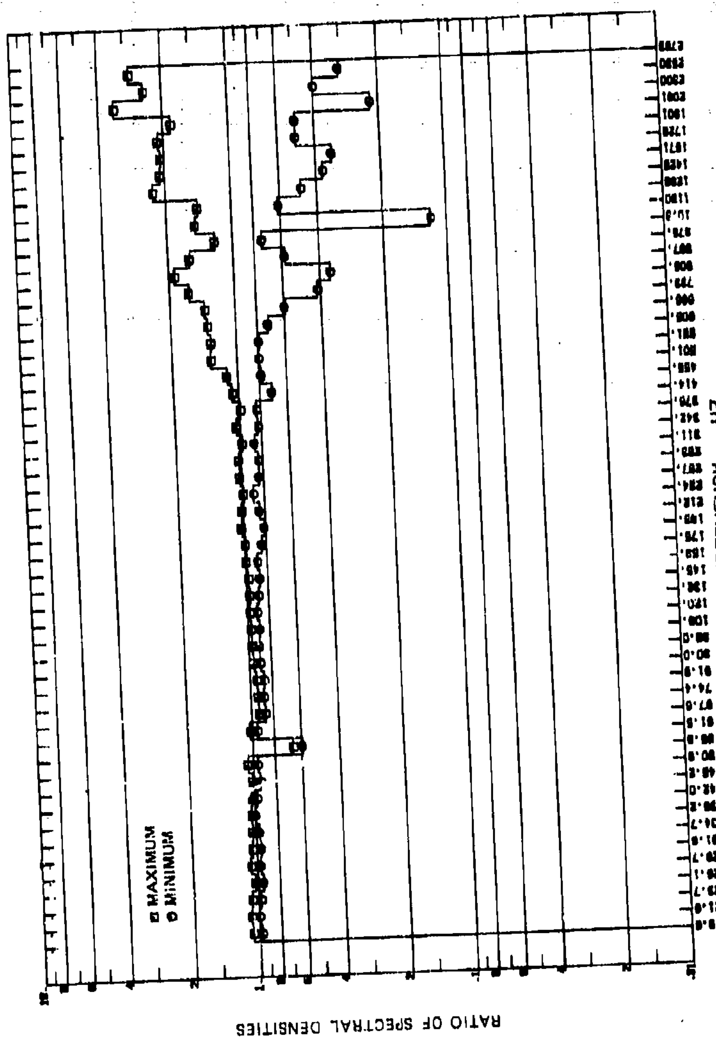

Fig. 2-4. Maximum and minimum variation of individual control accelerometer spectral density from power average of four accelerometers on fixture shown in Fig. 2.2 loaded with 40-lb unit.

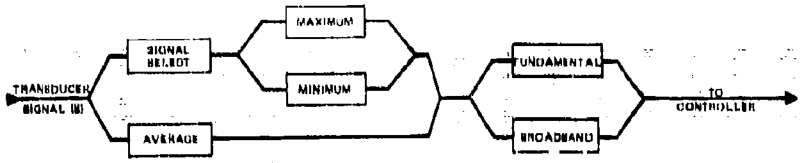

Fig. 2-5. Flow chart for selection of sinusoidal test level control.

requires a selection between using all the control transducer signals all the time or using only one signal at any particular time through a signal selection device. In the former case, the average of all transducer signals is obtained. In the latter case, the criterion for selection of a particular transducer signal must be established. Either the transducer that reads the maximum or the one that reads the minimum can be selected, as shown in Fig. 2-5.

It should be noted that by appropriate attenuation or amplification of individual transducer signals, the signal selection device can limit or transfer to different transducer signals at different physical levels, e.g., Channel 1 at 2 g, Channel 2 at 5 g, Channel 3 at 500 lb, etc. Similarly, weighted averages of the several signals could be obtained by modifying the inputs to the averaging device.

The next decision point requires the selection of the broadband signal or the fundamental, i.e., first harmonic of the broadband signal for control to the specified test level [31]. It should be noted that there is no technical reason why the two loops of Fig. 2-5 cannot be linked in reverse order. The order is, of course, trivial for broadband control. For fundamental control, reversal of the order strictly requires use of a tracking filter for each transducer signal, which may cause problems of equipment availability. Some of the considerations which enter into the selection of an appropriate method are discussed below.

The choice between averaging and maximum signal selection should ideally be based on the manner in which the test level itself was derived. If the test level represents extreme conditions, then maximum signal selection is appropriate. On the other hand, if the test level represents some kind of average level, or even a smoothed envelope of extreme conditions, then average control is appropriate. Test levels based on minimum conditions, if they exist, must be quite rare. However, even when the test conditions are based on extreme conditions, it may be desirable to use average control to insure that the intent of the test is not defeated because of an idiosyncrasy in one transducer signal due to rattling, a poor fixture, a poor transducer location, etc. Although difficult to substantiate, the argument can be made that the acceleration at one attachment point will generally be large in comparison to the other points only if it is relatively easy to create motion of that point, which also generally means that the motion will not be particularly damaging. Thus the real increase in severity by use of averaging

instead of signal selection may be less than the quantitative data implies, whereas the simulation of impedance characteristics may be improved.

Several Government test specifications such as MIL-STD-810B specify that the minimum transducer signal shall generally be selected for control unless massive test items are involved, in which case the average signal shall be used. It is believed the discussion at the beginning of this section refutes the propriety of the selection of the minimum signal.

A decision to select either the broadband or the fundamental (obtained by passing the broadband signal through a tracking filter slaved to the excitation frequency), whichever is chosen, may be difficult to justify. The need to make the choice arises from the inherent nonlinearities present in the vibration test equipment and the test object. These nonlinearities manifest themselves in the generation of harmonic distortion, particularly during excitation in the neighborhood of resonant frequencies. Test objects in which free play or clearances exist are notable in this regard. At certain frequencies, the peak-to-peak amplitude of the broadband signal may be as much as ten times the peak-to-peak amplitude of the fundamental component which is the only desired excitation.

Figures 2-6 and 2-7 are frequency plots obtained during a 2-g peak sinusoidal sweep of a unit weighing approximately 30 lb, installed in the fixture shown in Fig. 2-2. Vibration was applied in the lateral direction with the fixture on a slip plate driven by a 30,000-lb exciter. Figure 2-6 was obtained from the Accelerometer Signal Selector output, whereas Fig. 2-7 was obtained from the output of a response accelerometer mounted on the unit structure. In Fig. 2-6, the upper curve represents the rms acceleration at the fundamental or excitation frequency, and the lower curve represents the rms acceleration of all other components, i.e., the broadband signal minus the fundamental. In Fig. 2-7, up to 200 Hz, it is the lower curve which represents the fundamental frequency while the upper curve represents the distortion. The unit was characterized by a 30-Hz torsional resonant frequency which is prominent in the distortion of the input shown in Fig. 2-6. It is felt that these two figures illustrate the difficulties inherent in the selection of the sinusoidal control signal and, in addition, in the interpretation of the results of sinusoidal tests.

A literal interpretation of a test specification which called for a sinusoidal motion of a certain amplitude at or through a certain frequency range would demand that fundamental control be employed. On the other hand if fundamental control were used, then the amplitude of the harmonic distortion could well exceed the amplitude required by the specification in some frequency ranges, even though it did not occur at the time expected. Thus broadband control would, in a certain sense, engender some degree of undertest, while fundamental control would engender some degree of possible overtest. The majority of tests are performed using broadband control. It is not obvious whether this situation reflects a conscious decision that this type of control is more appropriate, is perhaps easier to "pass," or reflects the unavailability of suitable equipment for fundamental control.

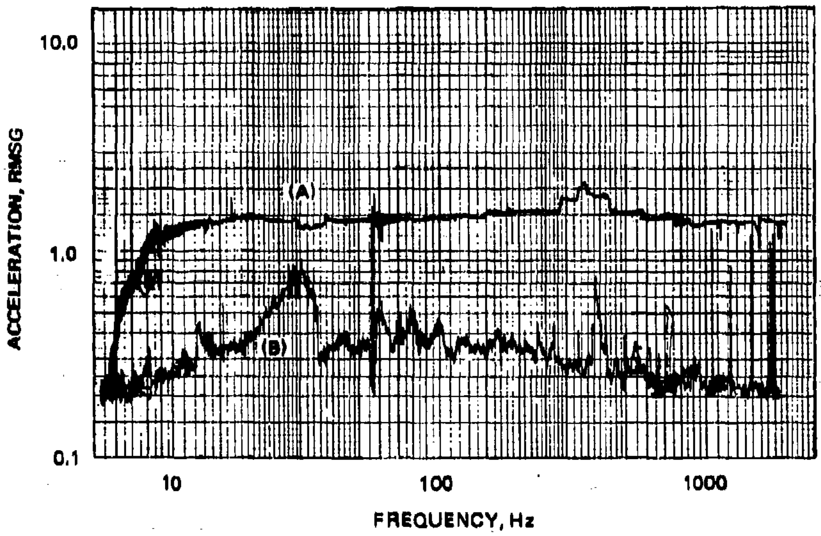

Fig. 2-6. Rms acceleration of control signal during swept sinusoidal test – (A) Fundamental and (B) Broadband minus fundamental.

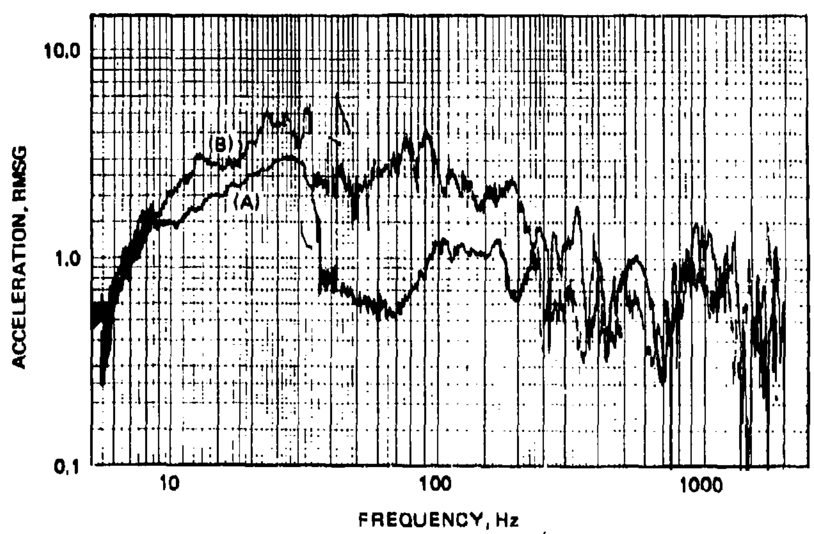

Fig. 2-7. Rms acceleration of response signal during swept sinusoidal test – (A) Fundamental and (B) Broadband minus fundamental.

Two situations where selection of fundamental control appear to be most appropriate are recognized. First, when frequency response functions are to be obtained from a sinusoidal sweep test, it appears to be more meaningful and, from a data reduction viewpoint, more economical. Granting that a frequency response function of a nonlinear system has meaning in the first place, it is more meaningful to obtain the response to a given constant level of sinusoidal excitation, especially if it is desired to obtain the response at a series of different levels. Second, if the input sinusoidal level can be held essentially constant through use of fundamental control, then the frequency response functions can be obtained by the use of a single tracking filter for filtering the playback of recorded response signals. The additional complexity of ratioing the outputs of two tracking filters (response/input) can often be avoided at small loss of accuracy. It should be clear to the reader at this point that it is of questionable value to obtain frequency response functions by ratioing the amplitudes of broadband response and input signals as measured from the envelopes of oscillographic recordings at slow paper speed.

Referring again to Fig. 2-5, the reader should be cautioned that interchanging the two loops of this figure when fundamental control is selected may present test implementation problems, which are discussed in Chapter 5. These problems arise due to the interrelationships between the time constants of the averaging or signal selection devices, the tracking filter, and the standard vibration equipment.

Random Test Level Control. Selection of the appropriate test level control method for random vibration testing is essentially trivial, although there is a choice of implementation of method. Assuming that multiple control transducers are to be employed, the only practical approach is to control the power average of the individual signals to the desired spectral density. The term *power average*, as opposed to the term *average* means that, within any narrow frequency band, the spectral density of the power average is equal to the average of the spectral densities of the individual signals. In other words, the mean square of the transducer signals is controlled. It is necessary, therefore, to synthesize a signal whose spectral density is equal to the desired power average of the individual signals. (The possibility of using a weighted power average by appropriate signal amplification or attenuation is self-evident.) Two means by which such a signal can be synthesized are discussed on page 118.

2.3 Data Requirements

Test specifications and procedures are not usually convenient vehicles for the definition of the data required during and after the completion of a vibration test. Although these data do not contribute directly to the definition of test conditions discussed so far, they are the means by which the satisfactory (or unsatisfactory) generation of the desired test conditions is documented and thus warrant some consideration here. These data can be grouped into three main categories.

1. Data taken for continuous monitoring during test
2. Data selected to verify test conditions
3. Data for engineering evaluation, e.g., response data.

The first category generally consists of continuous recordings of all control and some if not all response transducers. These data are acquired primarily for evaluation in case of failure in either the test object or the test equipment. Experience indicates that when a failure in the equipment under test occurs, it is a natural instinct for the designer of the equipment to question the test conditions in preference to questioning the adequacy of his design. In any case, it may be desirable to know the conditions at the time of failure. When a failure in the test equipment occurs, it is generally accompanied by a transient motion of unpredictable character. Thus it appears prudent to take raw data continuously during test even though most of it can be discarded almost immediately after conclusion of the test. Except for sinusoidal data for which oscillographic recording is generally adequate, magnetic tape recording is preferable, since later processing of the data is often required. The transducer signals to be recorded should be selected from a consideration of what information is needed to understand the conditions at any particular time, such as at failure.

The second category includes data selected by both the customer and the vibration test personnel. For data reduction, it is customary to select time samples of the transducer signals which are representative of the test conditions; e.g., spectral density plots, acceleration vs frequency plots, etc. The amount and kind of data required are determined by consideration of the minimum amount needed to adequately demonstrate that the specified test conditions were indeed generated or the degree to which they were not. Experience indicates that it is very easy to yield to pressure, in the name of time and economy, to practically eliminate this category of data but that, in the long run, it is false economy to do so.

The third category of data really has nothing to do with specification of the test conditions, assuming that a test method has been selected which has the potential to provide the desired data. However, a priori definition of these data may ensure that an appropriate test is selected. The amount and kind of data processing required to achieve the purpose of the test should be specified by the customer with the advice and consent of test personnel.

2.4 Necessary Accuracy

A lengthy discussion of the accuracy requirements to be specified for the performance of vibration tests is not appropriate to the theme of this monograph. The subject is not one which has received, in an integrated way, sufficient attention during the development of vibration testing. Small segments of the total problem have received inordinate attention, while other possibly more important segments have been largely ignored. As will be seen, accuracy requirements are often specified which are virtually meaningless due to lack of complete definition, and sometimes, due to physical impossibility.

This section has been intentionally entitled *necessary accuracy* rather than just *accuracy* in order to convey the thought that consideration must be given to specifying accuracy requirements which are compatible with the precision to which the nominal values are known, the precision with which the results of the test can be evaluated, and the value of achieving additional accuracy at additional test cost. In other words, the required accuracy is a factor in the design of the experiment. This is not to suggest that the normal good practices of using regularly calibrated instruments, proper calibration signals, etc., are unnecessary or should be relaxed. Rather it is to suggest that, for instance, there is little to be gained in requiring a resonant search using a sinusoidal sweep to take exactly N min. The only meaningful requirement is that it be slow enough, i.e., the sweep should be at least N min in duration.

The justifications often cited for specification of test parameters with rather small allowable variations are the need for repeatability of tests coupled with quality control requirements. These justifications presumably developed from experience that the same equipment tested at different locations, or different serial numbers of the same equipment tested at the same location, exhibited different responses and failures. However, experience also shows that even with very tight specifications, the variability of test results still persists, suggesting that the major variability in the results is due to parameters which have not been controlled and which probably cannot be either identified or controlled even if identified. The greatest contributor to such variability is the variation between nominally identical test objects. For example, Fig. 2-8 illustrates the variability in the squared transmissibility of five missiles during longitudinal vibration. The three curves represent the maximum, mean, and minimum values of the squared transmissibility, averaged over 10-percent bandwidths, between the forward and aft sections of the missile, a distance of approximately 8 ft. Without going into excessive detail, all factors except test object variability are believed to have been normalized out of these figures. Since the data of Fig. 2-8 were measured on the major structure and averaged over generous bandwidths, it is not hard to imagine the very large variability of responses of detailed parts, etc., within the electronic units mounted to this structure.

A second contributor to the variability of test results occurs when different test facilities are used. The use of different fixtures, different vibration exciters, and often different mounting locations of control transducers will all contribute to what, in effect, is a different test. Although the test conditions are nominally the same, the effects of different impedances in the two test configurations can well generate differences in test results.

There is little reason to expect that the variability in the service environment, upon which test levels are based, will be any less than those which are observed during test. In fact, there is good reason to expect considerably greater variation. Thus, while reasonable effort to maintain a certain accuracy in test condtions is necessary, it is suggested that only that precision essential to the purpose of the test be specified.

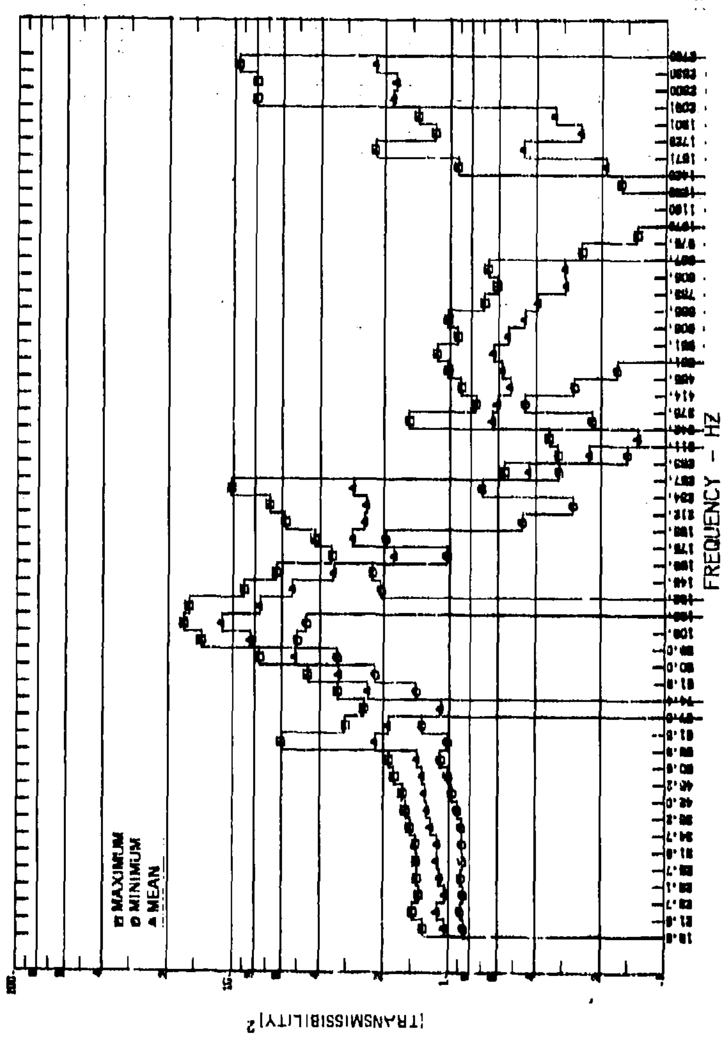

Fig. 2-8. Maximum, minimum, and mean squared transmissibilities of five missiles.

Six parameters which are significant, although not equally so, in the specification of tolerances during vibration tests are (1) duration, (2) frequency, (3) sinusoidal amplitude, (4) acceleration spectral density, (5) required equalization, and (6) fixture characteristics. Parameters 2, 3, and 5 appear in most specifications, e.g., MIL-STD-810B, whereas the sixth parameter has lately been included in a number of specifications for space equipment. A tolerance for the first parameter is seldom specified. Considerations which should enter into the selection of tolerances for those parameters are discussed below.

Duration

The specification of time enters into test specifications in two ways. First, the total test duration, or perhaps duration in each axis, is specified. Second, the time to accomplish some part of the test, such as a sinusoidal sweep, is specified. Of course time is, in a sense, the independent variable of the test, but nevertheless, should be permitted a reasonable specified variability. It is quite easy to control accurately yet is probably relatively unimportant to the overall test purpose. First, the derivation of the nominal test duration, as mentioned in the section on time duration (p. 33), is probably the most arbitrary test parameter. Second the shape of a typical fatigue curve is such that a 3-dB change in amplitude is equivalent to a factor of ten in time. Thus the efforts often made to set up a sinusoidal sweep so that it takes exactly 15 min, or to come back after repair of a failure to complete the last few minutes of a 3-hour test may be well meaning and satisfy specifications but hardly contribute to the overall value of the test program.

It is recommended that test specifications should generally include rather wide tolerances on durations so that undue efforts to meet the exact times now specified are avoided.

Frequency

A typical specification tolerance for "vibration frequency" is 1/2 Hz below 20 Hz or ±2 percent (MIL-STD-810B). It would perhaps be more logical to specify 25 Hz as the cutoff so that no step in the tolerance occurred. In any case, frequency in vibration testing is, like duration, more an independent variable than a controllable dependent variable. It is important to specify the accuracy with which it is measured but specification of a tolerance on frequency itself does not appear to be particularly meaningful. For example, in random vibration, only frequency bandwidths have meaning. When specifying the frequency range over which a test is to be conducted, e.g., 5 to 2000 Hz, it is appropriate to specify a tolerance on the upper and lower frequencies. In view of the manner in which these bounds are selected, as discussed in the section on frequency range (p. 22), it is perhaps questionable whether the tolerance need be as tight as ±2 percent.

Sinusoidal Amplitude

Again citing MIL-STD-810B as typical, the tolerance for sinusoidal tests is "Vibration Amplitude: Sinusoidal ±10 percent." This statement should immediately raise the question of whether the tolerance applies to the broadband peak amplitude or to the amplitude of the fundamental or perhaps to the rms value of the broadband signal. Test conditions which comply, using one of these three possibilities, would almost certainly violate the same tolerance applied to the other two. Thus the quantity to which the tolerance applies must be clearly identified. A specified tolerance, if it is to be meaningful and not ignored, must be unambiguous and attainable with reasonable effort and cost. Experience indicates that the specification of a fairly small percentage variation, such as ±10 percent, across the entire frequency range of test is often not attainable with reasonable effort, particularly during sweep tests. This is due to the interaction of the control system and the reflected load of the test object and, when a single control transducer is used, the occurrence of nodes at the control transducer location. Of course, the inability to meet the specified tolerance will be determined only at the time of test and cannot be determined a priori. At this point, calling a halt to the test program in order to attempt to comply tends to be traumatic as well as fruitless. A more reasonable approach, which has been used on a number of occasions, is to specify a tolerance, such as ±10 percent, which must be maintained over most of the frequency range and a much wider tolerance, such as +100, -50 percent, which must be maintained over the remainder. For example, one could permit variations in excess of 10 percent in several narrow frequency bands, each no wider than say 1/10 of an octave with a cumulative bandwidth of say 1/3 of an octave over which the variation exceeds 10 percent. Such a requirement is reasonably attainable, does achieve the objective of avoiding a poor quality test, and will therefore be complied with.

Spectral Density

Regardless of the data processing method employed, a measurement of the spectral density of a random process has two equally important characteristics which should be included when specifying a tolerance about some nominal value. First, any measurement represents the average spectral density of the signal within the analysis bandwidth. (It should be noted that the concept of analysis--or effective--bandwidth is based on the contribution of the skirts to the output of the filter when a white noise input signal, i.e., constant spectral density, is applied to the filter. References 32 and 33 describe the smoothing effects when varying spectral density signals are applied.) Thus the specification should include a statement regarding the maximum acceptable analysis bandwidth to be employed. Second, apart from measurement or analysis inaccuracy, any measurement of spectral density is subject to statistical or sampling error. This error is normally defined by

$$e = 1/\sqrt{BT}, \tag{2-1}$$

where e is the normalized standard error, B is the analysis bandwidth, and T is the sample data length. Reference 33 discusses this error in detail. From Eq. (2-1), it is seen that a tolerance on spectral density must define a minimum acceptable BT product. The interaction of these two characteristics is self-evident. To this writing, practically no specifications for random vibration include a statement regarding both these characteristics.

Required Equalization

As described in detail in Chapter 5, equalization is the term used in random vibration testing to describe the shaping of the output spectral density of a noise source to produce the desired test spectrum at the control point or points. The noise signal is amplified or attenuated within contiguous bandwidths of a comb filter bank, while the achieved spectrum is monitored through an identical comb filter. The bandwidths and analyzer and servoamplifier time constants of each equalization channel must be chosen with due regard for the statistical errors discussed under spectral density in Section 2.4. Making the generally safe assumption that the equalization equipment manufacturer has made a proper choice, it remains to specify the tolerance on the achieved spectral density. It is common practice to specify a tolerance of ±3 dB (+100, -50 percent on spectral density) across the frequency range or alternatively, to specify ±1.5 dB (+40, -30 percent) below 1000 Hz and ±3 dB above 1000 Hz. The latter practice recognizes the relatively easier task of achieving the required values at lower frequencies. Compared to the typical ±10-percent tolerance on sinusoidal amplitude discussed previously, these are generous tolerances which probably reflect early random vibration test experience when the equalization process was carried out manually, i.e., with human servos.

An additional requirement that the overall rms acceleration, i.e., the square root of the area under the curve, be maintained within a certain tolerance, say ±10 percent, is often included. Presumably this prevents unscrupulous testers from running the test at -3 dB across the whole frequency band. Three problems arise in using tolerances specified as above. First, the maximum bandwidth within which the tolerance shall apply is unspecified. Second, it is frequently impossible to meet the requirements over part of the frequency range. Last, inappropriate methods of demonstrating compliance are specified.

To expand on these problems, consider the first one. The comb filters of most commercially available equalizer/analyzer systems have bandwidths which increase from about 10 Hz centered at about 15 Hz to constant values of 25, 50, or 100 Hz, depending on the number of channels in the system. Unless specified to the contrary, a particular test may be conducted using any of the above filter banks and the tolerance on spectral density will be observed for each channel of the analyzer section. Remembering that the measurement is the average spectral density in the analysis bandwidth, it is clear that very different but nominally identical tests can be performed by changing equalizer systems. Therefore, if

there is adequate reason to select a particular bandwidth for control, the equalizer characteristics must be specified a priori.

The second problem is similar to the situation discussed under sinusoidal amplitude on page 44. A similar modification which permits larger deviations over restricted bandwidths has been found satisfactory. For example, a specification might state that a larger variation of no more than $\pm x$ dB in y equalizer channels is permissible. In random vibration testing, the generation of harmonic distortion due to nonlinearities, discussed earlier for sinusoidal testing, will cause excessive responses in one analyzer channel due to the excitation in a different channel of the equalizer. Attenuating the noise input in the channel with the excessive response is obviously useless, and identifying the channel which is the source of the response is impossible. For random vibration testing, the occurrence of nodes at control transducer locations is more significant than for sinusoidal testing due to the broadband nature of the signal. If automatic equalization is employed, the shaker system will attempt to overcome the effects of the node by demanding a large amount of power in the frequency range of the node. Limitations of both dynamic range and available power will then be reached before the desired level is achieved across the whole frequency range. Therefore it is necessary to identify the frequency band of the node and depress the input in this band so that the proper level is reached elsewhere. This can usually be accomplished by making a spectral analysis of the motion of the shaker head. It appears reasonable that a test at the desired level over 90 percent of the frequency range is more useful than a test at a depressed level over the entire frequency range. Thus a specification tolerance of the type suggested above will achieve the objective of the test and can reasonably be achieved.

The last of the three problems, namely the specification of inappropriate methods of demonstrating compliance, is more a philosophical problem, even though the severity of its impact on the ability to conduct a timely and useful test program is difficult to describe and is almost unbelievable. With some assistance from Murphy's Law,* an inappropriate demonstration method will almost certainly show that excessive deviations occurred. The problem is basically one of frequency resolution combined with a confusion of the desires to achieve a certain test spectrum and then know what spectrum was achieved.

The root of the problem is illustrated in Fig. 2-9, which depicts five equal contiguous bandwidths (B). If a signal whose actual spectral density is shown by the dotted line is analyzed with five contiguous ideal filters, with bandwidths equal to and lined up with those in Fig. 2-9, then each measurement will be identical. Each filter measures the average spectral density within the bandwidth, and the shaded areas above and below the horizontal line in each bandwidth are equal. If the center frequency of one of these filters is adjusted, as shown by the dotted lines, the measured spectral density will change as shown, since the average over a different bandwidth is now obtained. If a different but still ideal

*Anything that can go wrong will.

filter with bandwidth B' is used to measure this same signal, it will measure different values than the filters with bandwidth B as shown in Fig. 2-9. The continuous spectral density plots obtained from a swept-frequency analyzer tend to obscure the fact that each point on the curve actually represents a value averaged over the analyzer bandwidth. A comb filter permits the more graphic bar-chart display such as Fig. 2-8, where the averaging bandwidth is represented by the width of each bar.

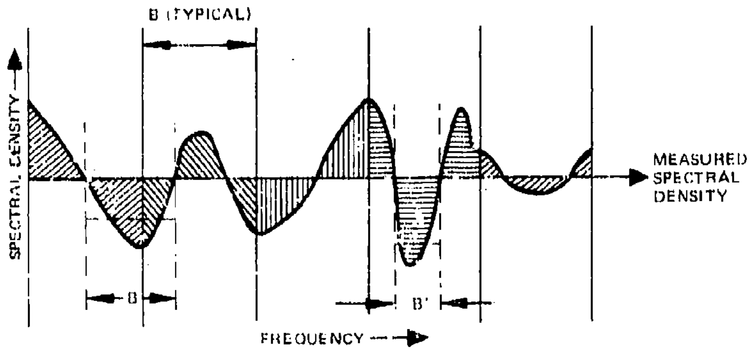

Fig. 2-9. Apparent or measured spectral density vs actual spectral density.

When the effects of using real fiters, each with its own characteristc deviation from an ideal filter, and when it is remembered that the crossover frequencies between adjacent filters in a comb filter usually occur at the half-power (−3-dB) points of the filter, it should not be surprising that two spectral density plots describing a single signal will differ considerably in fine detail unless the same bandwidths and center frequencies are used in each analysis.

If the spectral density of a test level control signal is determined with an analyzer, particularly a swept-frequency analyzer whose bandwidth is equal to or less than the filter bandwidths of the equalizer/analyzer system used to control the test, it is quite likely that such an analysis will indicate that the spectral density exceeded the allowable tolerance, particularly in the frequency region above 1 kHz. This will happen even though that same record, played back through the equalizer/analyzer system, will show that the required spectrum was achieved. It is suggested that this problem is resolved by recognizing the following: First, when a test plan is approved, tacit approval of the frequency resolution of the equalization system is included in the approval. If a particular bandwidth equalizer is required, it must be specified at the time. Second, the approval also implies that the tolerance on spectral density is to be achieved and demonstrated using the equalizer/analyzer bandwidths. Third, any later analysis of the control signal with different analysis bandwidths is performed merely to find out what happened and not to attempt to control the test. It is believed

that any other approach merely leads to fruitless arguments since it is patently impossible to control the spectral density in one bandwidth with a filter which has a different bandwidth. Last, if one considers the basic purpose, the simulation aspects of the test, and the data from which the test level was derived, it is probably desirable to control the average spectral density, particularly above 1 kHz, over bandwidths which are at least as wide if not wider than incorporated in most common equalizer/analyzer systems.

Fixture Characteristics

A trend has become evident during the past year or two in which random vibration test specifications for units to be tested prior to installation in space vehicles have included requirements on the characteristics of vibration fixtures. The design of suitable fixtures, discussed in more detail in Chapter 4, is a continuing source of difficulties in vibration testing. The sources of some of these difficulties have been mentioned in previous sections. One difficulty not previously mentioned is one of economics. Specifically, insufficient time and money are generally allocated to the design and fabrication of test fixtures relative to the total cost of the test, particularly when one recognizes the influence that test fixture characteristics can have on test results. Assuming appropriate location of control transducers, inadequacies of test fixtures generally manifest themselves as either the inability of the shaker system to produce the desired test level or the cause of unrealistic failures in the test object due to overtest. Undertest is also possible but generally is much less likely. Specification of required characteristics of test fixtures is directed toward eliminating these types of problems. Like most things, however, carrying this to extremes can create problems worse than the original one.

These specifications on fixtures generally contain three requirements. The first one is that the adequacy of the test fixture be demonstrated with the real test object mounted in place. Second, the specified variation, or rather lack of variation between motion at the attachment points of the unit is to be demonstrated by making a low level, 1-g or 2-g, sinusoidal sweep through the frequency range. Third, the permissible variation between any two attachment points is to be limited for example to 6 dB, or a factor of two in amplitude, over a frequency range of 20 to 2000 Hz.

The following implications of this kind of requirement should be considered. If the demonstration is to be conducted with the test unit in place, it can only be performed just prior to the real test. Since the type of programs in which these requirements have appeared are usually characterized by very tight schedules, the discovery of a fixture inadequacy at this time tends to cause a certain amount of anxiety. Since it is the fixture characteristics which are to be examined rather than the impedance effects of the unit, it is suggested that either the empty fixture or the fixture loaded with a simple dummy mass might serve the purpose equally well.

This suggestion also assists in resolving the following problem. It is desired that the fixture be adequate during a randome vibration test, generally at high spectral density levels. Because of nonlinearities, the variations measured during a low level sinusoidal sweep and those measured during test will be quite different, with much greater variation during the former. Thus the fixture should be evaluated at full level which is possible either empty or with dummy load but not with the real unit. This approach also avoids the problem, discussed on page 36, of deciding how to account for harmonic distortion during the sinusoidal sweep.

The manner in which permissible variations are to be specified requires careful consideration. First, it is the variation between the motion at any attachment point and the motion represented by the control signal which is important, whether this be a point on the fixture somewhat removed from an attachment point or a power average signal (see Section 2.2, discussion of random test level control) derived from the motion at several attachment points. Second, the maximum permissible variation should be described in a manner which is physically achievable. Except for very compact test objects, this means that large variations must be permitted over some reasonable frequency range, similar to the suggestions made in previous discussions of equalization and control tolerances. Of course, the fixture is only one link in the shaker/fixture/test object system which is to be controlled within some acceptable tolerance band.

To illustrate the problems of fixture specification, consider the fixture sketched in Fig. 2-10 which is the plan view of a 3-1/2-in. thick aluminum plate which bolts directly to the head of a 30,000-lb exciter. It was used for testing several light units together as an operating subsystem. One of the units was rectangular, approximately 15 × 25 × 5 in. thick, and spanned most of the fixture as shown. This unit weighed approximately 25 lb while the fixture weighed 200 lb. The unit attached to the fixture and to the spacecraft by 21 No. 10 screws. To evaluate the fixture, accelerometers were attached in the rectangular grid shown in Fig. 2-10. Acceleration spectral density plots were obtained for each accelerometer signal during random excitation of the empty fixture at $0.2g^2$/Hz between 20 and 2000 Hz controlled at location A. The spectral analysis employed a 10-percent bandwidth. Using a digital computer program, these spectra were examined to determine preferred locations of units on the fixture and preferred combinations of accelerometers for power average control, which would minimize variations between unit attachment point and control signal motions. Below approximately 800 Hz, the fixture behaved essentially as a rigid plate. The variations above 800 Hz are illustrated in Fig. 2-11. The two curves of this figure represent the maximum amplification and maximum attenuation between any accelerometer location in Fig. 2-10 and accelerometer location B. The values were obtained from the square root of the ratio of the spectral density values in each bandwidth. In other words, they

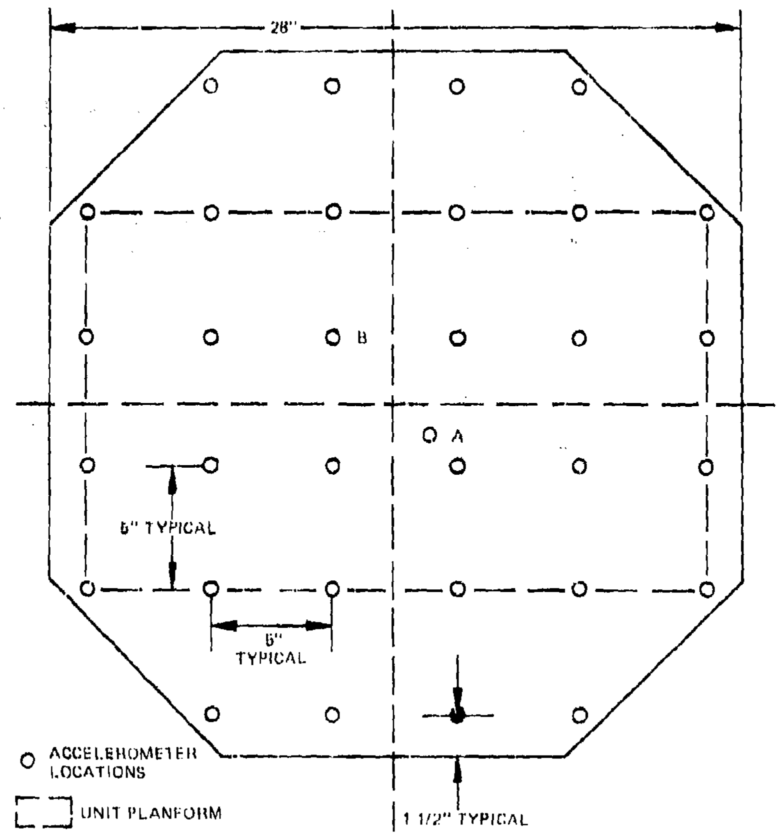

Fig. 2-10. Flat plate fixture, 3-1/2-in.-thick aluminum.

represent approximately the rms transmissibility within each 10-percent bandwidth. In decibels, the largest deviations were +18.6 dB and −14.9 dB. While this example may be a 3σ situation, it is believed that the need for care in applying fixture specifications is apparent. Clearly, a more simple or rigid fixture is hard to imagine and the units have to be tested in whatever size they are built. Although it might be desirable, specifications so far have been unable to change the laws of physics.

Besides consideration of specifying fixture characteristics which are practical to achieve by use of normal vibration test equipment and good engineering practice, consideration should also be given to the effects of such a specification on the simulation characteristics and therefore confidence in the test results. The kind of test objects cited at the beginning of this discussion typically mount to spacecraft structure through many small screws. It is suspected that in many

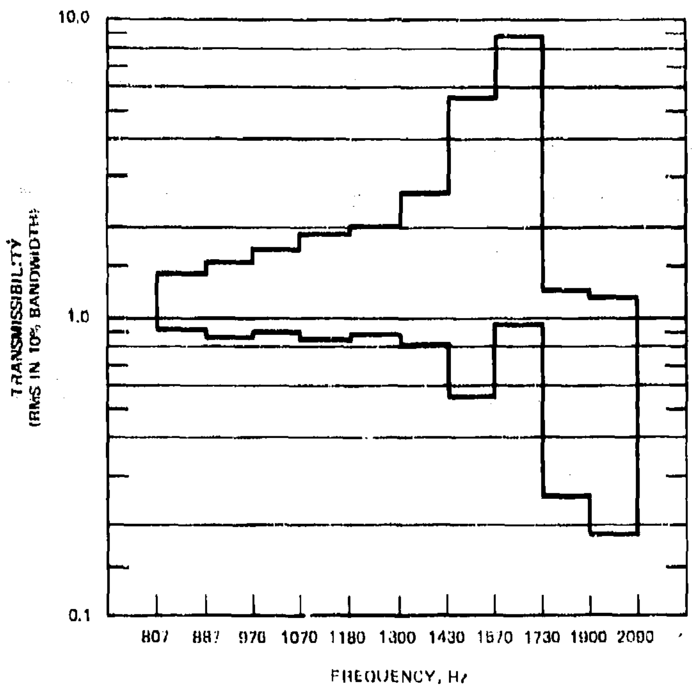

Fig. 2-11. Maximum amplification and maximum attenuation of vibration fixture (see Fig. 2-10).

instances, it is the unit which stiffens the structure, rather than vice versa. Testing of such units on very rigid fixtures, such as a 3-1/2-in.-thick magnesium plate, is less than a complete simulation and obviously introduces atypical responses in the unit during test. While it is still beyond our capability to solve this simulation problem, it does not appear that encouraging a trend to more rigid fixtures through overly rigid fixture specifications is a step toward improved simulation.

Summary

The discussion in this section under the title of "Necessary Accuracy" has been rather far ranging and has attempted to analyze a number of factors which, when controlled to appropriate tolerances or accuracy, will provide useful and valid test results. In effect, an approach to specification of vibration tests which constitutes a balanced design of experiment has been described. It is

perhaps appropriate to conclude the discussion by summarizing the main points which were as follows:

1. Selection of permissible variations of test conditions should be based on requiring only that accuracy necessary to achieve a proper design of experiment.

2. The allowable tolerance specified on vibration level should be consistent with the accuracy with which the level was derived.

3. If specified tolerances are to be useful, their specification must be complete and unambiguous.

4. If specified tolerances are intended to be complied with, the tolerances must be physically attainable with reasonable engineering effort.

5. The use of power averaging control in random tests and averaging or signal-selection control in sinusoidal tests will both improve the quality of the test and the ease with which specified conditions can be achieved.

6. The quality and value of a vibration test is primarily a function of the competence of the test engineering personnel. The use of close tolerances as a means of achieving high-quality tests is not always effective and may even be detrimental.

CHAPTER 3
SIMULATION CHARACTERISTICS OF TEST METHODS

The purpose of most vibration tests is to simulate the conditions that will occur in the intended use of an item, i.e., its vibration environment. It was stated earlier that tests are intended to simulate either the environment or its effects. In reality, vibration tests only simulate the effects. Real environments are much too complex to reproduce exactly; in addition to the factors such as waveform, impedance, excitation direction, etc., there are the effects of other environments which may act simultaneously with the vibration such as high temperature, acceleration, etc. Hence, vibration tests are designed to simulate the more important vibrational characteristics of actual conditions and thus produce the desired effects. The important characteristics are related both to the objectives of the test and to the damaging effects of vibration. These factors, which are interrelated, dictate the degree of simulation required. Chapter 2 contains a discussion on the simulation characteristics of various test parameters and techniques in terms of how these factors relate to actual environments and their importance in relation to the objective of the test. Conversely, this chapter contains a discussion of simulation characteristics in terms of effects. The test parameters of the standard methods, i.e., sinusoidal dwell, sinusoidal sweep, and broadband random, are examined and compared on the basis of their damage potential. In addition, the characteristics of two "non-standard" methods, narrowband random and gunfire vibration, are briefly discussed.

The discussions of test simulation here are, of necessity, more mathematical than other chapters although the treatment has been purposely simplified. More rigorous treatments of the material are found in works listed in the bibliography. A word of caution: The subject of vibration is complex and the simplified mathematical treatment opens the door to misinterpretation and misapplication. Even if the mathematics is manageable there is the danger of ascribing more accuracy to the analytical results than is justified by the accuracy of the input data.

3.1 Mathematical Model for Measurement of Simulation Parameters

Where necessary, the properties of different test methods or of a single test method as a function of the test conditions are compared by measuring response parameters of a single-degree-of-freedom (SDF) system. The equations for an SDF system are convenient and familiar to most engineers. They are not limited, however, to the study of SDF systems, since in normal mode theory the differential equations of motion for a single normal mode of a multidegree-of-freedom system have the same form as those for an SDF system.

The mathematical model used for the evaluations is shown in Fig. 3-1. A base-excited system was chosen because the preponderance of tests are performed by controlling the base motion. The differential equation of motion for this system is

$$m \frac{d^2 x}{dt^2} + c\left(\frac{dx}{dt} - \frac{ds}{dt}\right) + k(x - s) = 0,$$

or, alternatively, in terms of the relative motion,

$$m \frac{d^2 y}{dt^2} + c \frac{dy}{dt} + ky = - m \frac{d^2 s}{dt^2} ,$$

where

m = mass (lb-sec^2/in.)
c = viscous damping constant (lb-sec/in.)
k = spring constant (lbs/in.).

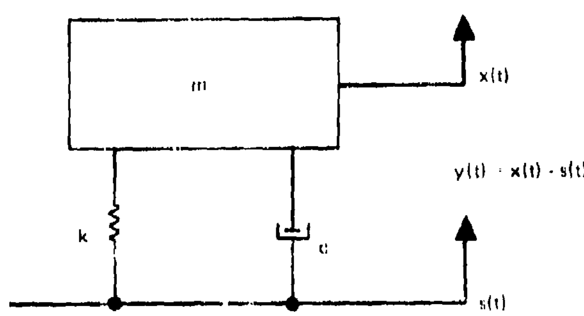

Fig. 3-1. Mathematical model of single-degree-of-freedom (SDF) system.

The steady state solutions of these equations for sinusoidal excitation of the form $s(t) = S_0 \sin \omega t$, where both S_0 and ω, the forcing frequency, are constant with time are

$$\frac{x}{S_0} = T \sin(\omega t - \theta),$$

and

$$\frac{y}{S_0} = H \sin(\omega t - \theta),$$

where T and H in these equations are defined as the motion transmissibility and the amplification factor, respectively. The equations for these factors are

$$T = \sqrt{\frac{1 + (2\zeta\omega/\omega_n)^2}{\left[1 - (\omega^2/\omega_n^2)\right]^2 + (2\zeta\omega/\omega_n)^2}} \tag{3-1}$$

and

$$H = \sqrt{\frac{1}{\left[1 - (\omega^2/\omega_n^2)\right]^2 + (2\zeta\omega/\omega_n)^2}}\ , \tag{3-2}$$

where

ζ = fraction of critical damping ($c/2\sqrt{km}$) (dimensionless)
ω_n = undamped natural frequency ($\sqrt{k/m}$) in rad/sec
$f_n = \omega_n/2\pi$ Hz.

The transmissibility T is plotted vs the nondimensional frequency ω/ω_n in Fig. 3-2 for several values of damping.

For steady state excitation the maximum response will occur for the forcing frequency approximately equal to the natural frequency, $\omega/\omega_n = 1$. In this case Eq. (3-2) reduces to

$$H = \frac{1}{2\zeta}\ .$$

The value $1/2\zeta$ is defined as the peak amplification or quality factor and is commonly referred to as Q.

The quantity Q, a term often used in electrical engineering, is a measure of the sharpness of the resonant peak of an SDF system. This is illustrated in Fig. 3-3, which is a detail of the resonance area of a response vs frequency curve. The bandwidth B of this resonance peak measured at the half-power point (i.e., at a value $R = R_{max}/\sqrt{2}$) is approximately related to Q by

$$B = \frac{\omega_n}{Q} \tag{3-3}$$

for values of damping, ζ, less than 0.1.

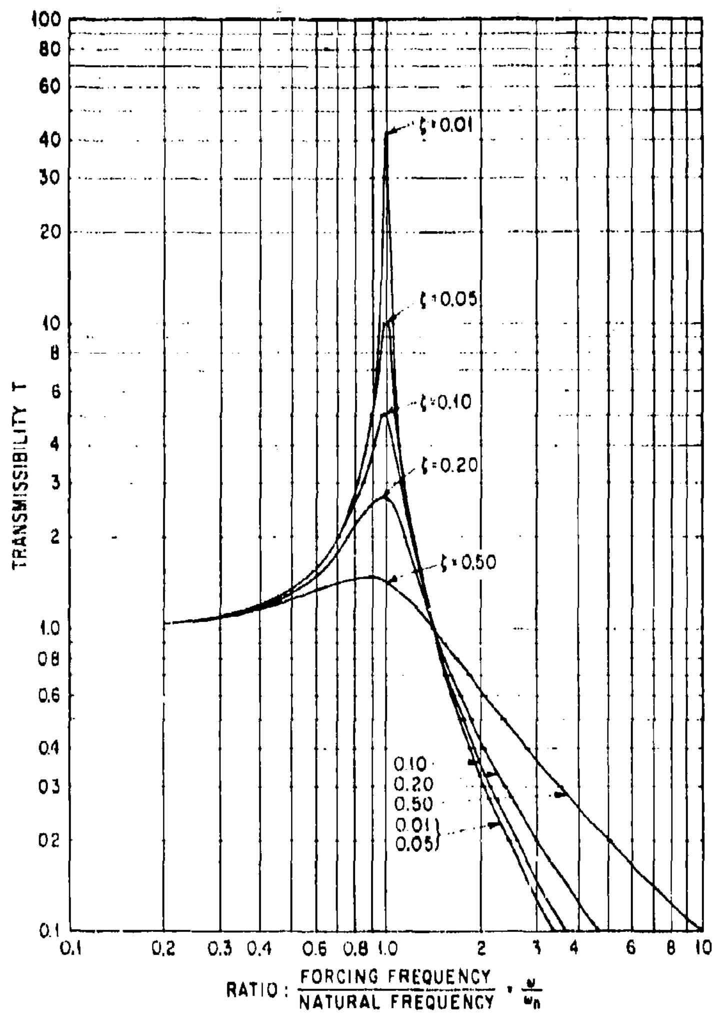

Fig. 3-2. Transmissibility functions of SDF system (from Eq. 3-1). From *Shock and Vibration Handbook*, vol. 1, Fig. 2.17, p. 2-12; copyright 1961 by McGraw-Hill Book Co., Inc. Used by permission.

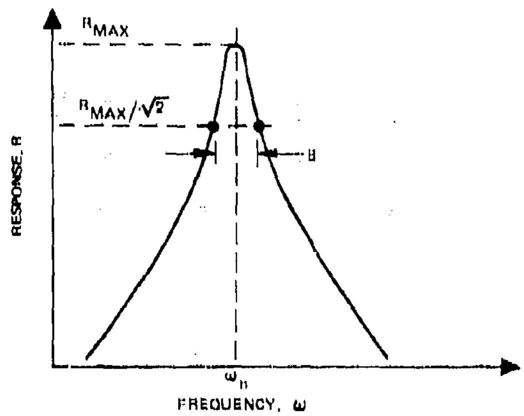

Fig. 3-3. Resonance area of response vs frequency illustrating half-power bandwidth (B). From *Shock and Vibration Handbook*, vol. 1, Fig. 2.22, p. 2-15; copyright 1961 by McGraw-Hill Company, Inc. Used by permission.

3.2 Sinusoidal Test Methods

Single-Frequency Sweep

Of the two standard sinusoidal methods, the single-frequency sweep is the least likely to resemble an actual environment. Nevertheless, it is a more desirable test than the resonance dwell for reasons which are explained in the discussion of resonance dwell testing (page 67).

In the sweep test the excitation frequency ω is continuously varied between an upper and a lower frequency limit. The rate of change of the excitation frequency and the method of varying this rate as a function of test frequency have a significant effect on the response of equipment. The sweep rate controls the amplitude of resonant response, and the sweep method controls the amount of time or number of cycles in any frequency range.

Effect of Sweep Rate. When a specimen is excited by constant sinusoidal excitation at a resonant frequency, the amplitude of the response will gradually build up to a level proportional to the level of excitation and the amplification factor of the resonance. This final level is termed the steady state response. The number of cycles of constant excitation required to obtain steady state response is proportional to the amplification of the resonance; the greater the Q the more cycles necessary to build up to steady state response. When the excitation frequency is varying, as in a sweep test, the number of cycles in any frequency band is dependent on the rate of change of excitation frequency. Steady state response can be approximated only if this rate is slow enough to allow a sufficient number of cycles to occur in the bandwidth of the resonance (see Fig. 3-4).

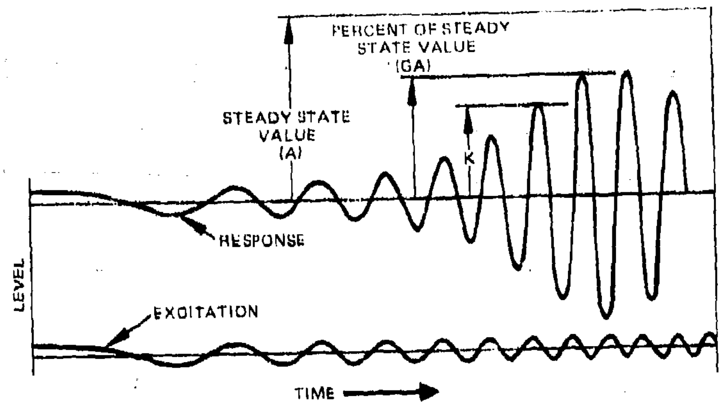

Fig. 3-4. Excitation and response time histories of sweeping sinusoidal inputs.

Exact mathematical solutions [34-36] of the response of a linear SDF system to a sinusoidal vibration whose frequency is varying are complicated and dependent on several variables, such as sweep rate, damping, natural frequency, sweep method, and direction of sweep. As a good approximation it was found [37] that the percentage of maximum steady state response is dependent on a single parameter which combines damping, resonant frequency, and the time rate of change of the excitation frequency, as the excitation passes through the resonant frequency. The fraction of steady state response vs this sweep parameter is shown in Fig. 3-5. In terms of the sweep parameter η, the fraction of steady state response G is approximated by

$$G = 1 - \exp\left(-2.86\eta^{-0.445}\right), \tag{3-4}$$

where

$$\eta = \frac{Q^2}{f_n^2}\,|\dot{f}| = \frac{|\dot{f}|}{B^2} \text{ (sweep parameter)} \tag{3-5}$$

$|\dot{f}|$ = absolute value of time rate of change of frequency at resonant frequency f_n

B = half-power bandwidth.

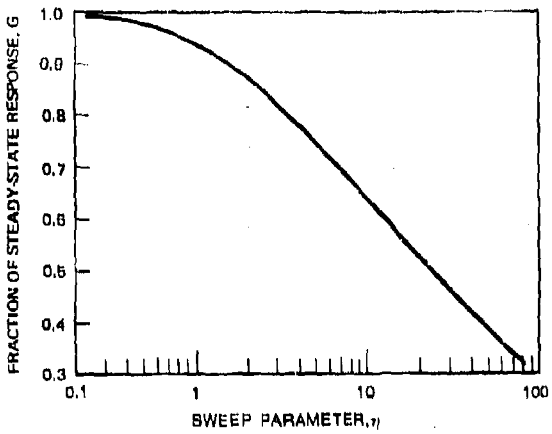

Fig. 3-5. Fraction of steady state response attained by a mechanical oscillator as a function of sweep parameter η (from Ref. 37).

With a sweep parameter of one, the response obtained in a sweep test will be approximately 95 percent of the value which could be obtained in a dwell at the resonant frequency.

In addition to the effect on response amplitude, the sweep rate has an effect on the frequency of peak response. With an increasing excitation frequency the peak response will occur at a frequency greater than the resonant frequency. With decreasing excitation frequency the peak will occur at a frequency less than the resonant frequency. The amount of shift, like the amplitude of response, is dependent on sweep rate, damping, etc. This effect is illustrated in Fig. 3-6, which shows a series of response curves at various sweep rates for both decreasing and increasing excitation frequency [34].

Effect of Sweep Method. Depending on the purpose of the test, one may wish to control the number of cycles or the time at high amplification. For example, simulation of transients may require an equal number of cycles, since natural modes with equal damping will decay in an equal number of cycles. Similarly, a fatigue life-test would dictate an equal number of cycles at each resonance. However, for a service life-test equal time at high amplification may be more desirable. The appropriate sweep method will be different for each requirement, and may be determined as outlined in the following paragraphs.

Sweep Method to Produce Equal Number of Cycles at Each Resonance. For a system excited by swept frequency, the number of cycles at any resonance with peak levels equal to or greater than a given fraction K of steady state response (see Fig. 3-4) can be approximated by

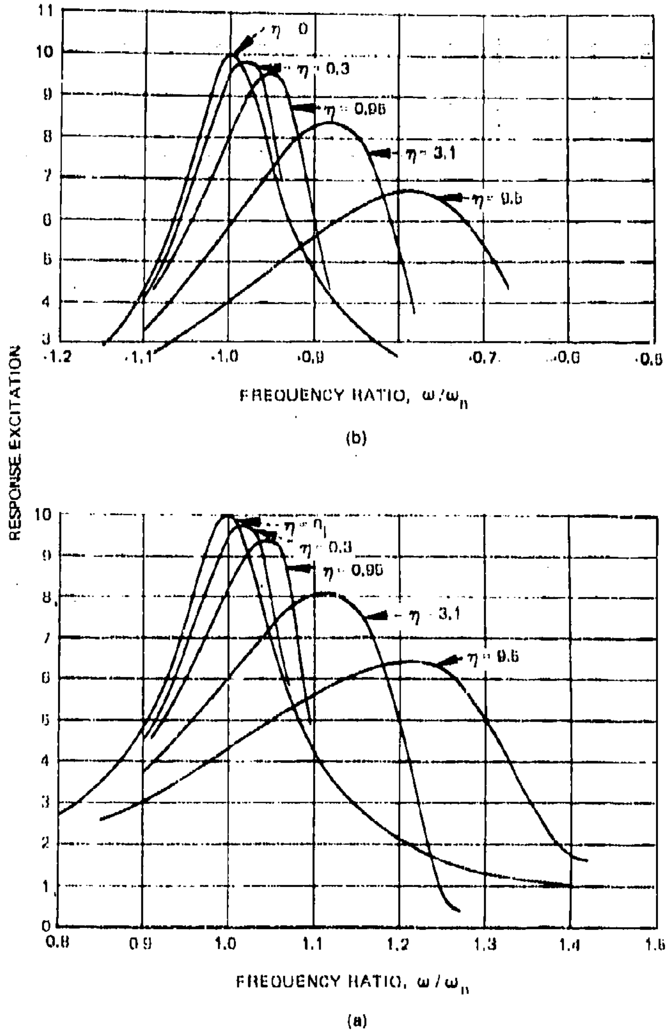

Fig. 3-6. Resonance envelopes of an SDF system subjected to sinusoidal sweep ($Q = 10$) for various values of sweep parameter η (from Ref. 34): (a) increasing frequency and (b) decreasing frequency. Reprinted from "Vibrations During Acceleration Through a Critical Speed," by F. M. Lewis, *Trans. ASME* 76, pp. 253-261; copyright 1932 by the American Society of Mechanical Engineers. Used by permission.

$$N_i = \frac{Q_i\sqrt{1/K^2 - 1}}{\eta},$$

where the subscript i refers to the ith resonance. Substituting for η from Eq. (3-5) results in

$$N_i = \frac{f_i^2\sqrt{1/K^2 - 1}}{Q_i|\dot{f}|}. \quad (3\text{-}6)$$

For an equal number of cycles at each resonance of a specimen, it is evident from Eq. (3-6) that the sweep rate must be inversely proportional to the amplification factor and proportional to the square of the frequency. That is,

$$|\dot{f}| \propto \frac{f_i^2}{Q_i}.$$

A relationship between Q_i and f_i would allow formulation of a proportionality between sweep rate and sweep frequency. There are two types of damping, system and material, which must be considered in searching for a relationship. System damping includes the damping that occurs in (1) interfaces, joints, and fasteners, (2) electromechanical systems where energy is dissipated because of interaction between electrical or electromagnetic phenomena and physical bodies, and (3) hydromechanical and acoustic systems where energy is dissipated through fluid flow. Material damping, conversely, involves the energy which is dissipated within the body of the structural material. System damping, even though it can be an important mechanism in the total energy dissipated by a specimen, does not lend itself to mathematical treatment and is therefore not discussed here. Material damping, on the other hand, has received considerable mathematical treatment with results that have important significance for vibration testing. Lazan [38] shows that the amplification of a natural mode of a specimen is related to the total strain energy W_0 and energy dissipated D_0 in the mode by

$$Q = \frac{W_0}{D_0}.$$

The strain energy is proportional to the square of the stress, which, in turn, is proportional to the vibration response (e.g., deflection velocity, or acceleration). The specific damping energy is related to the stress by

$$D = J\sigma^n,$$

where J is a material constant. This relationship is approximated from damping vs stress data gathered from a variety of materials as illustrated in Fig. 3-7. For viscoelastic materials $n \neq 2$. For many structural materials, however, $n \neq 2$ and has representative values of $n = 2.4$ in the low-to-intermediate stress regions (stress levels less than 80 percent of the endurance limit) and $n = 8$ in the high stress regions. The total damping energy D_0 is related to the specific damping energy D by factors which account for specimen geometry and stress distribution. The equation for Q is therefore restated as

$$Q = K\,(\text{response level})^{2-n}, \tag{3-7}$$

where K is a constant only for a given material, specimen geometry, and stress distribution. Consequently its value will vary widely for different natural modes of a specimen. Therefore, for most systems, there is no valid relationship between Q and natural frequency, and the sweep rate proportionality cannot be reduced to a function involving sweep frequency alone.

There are, however, certain special cases where Eq. (3-7) is useful for defining a sweep method. For viscoelastic materials $n = 2$ and, from Eq. (3-7), Q is independent of response level. In this situation ($n = 2$), the constant K is independent of specimen geometry and stress distribution and is a function of material alone. Therefore, if the damping is dissipated in the same material for all modes, Q is a constant, and a sweep rate proportional to the square of the sweep frequency will produce an equal number of cycles in all resonances of the specimen.

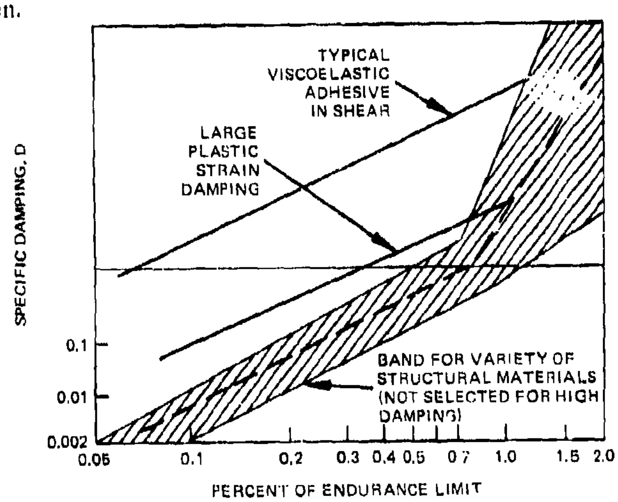

Fig. 3-7. Damping stress relationships (from Ref. 38). From *Structural Damping*, papers presented at a colloquium at the ASME annual meeting in Atlantic City, N.J., December 1959; copyright 1959 by the American Society of Mechanical Engineers. Used by permission.

Sweep Method to Produce Equal Time at Each Resonance. The time spent in any frequency band is equal to the number of cycles of oscillation that occurred while in the band divided by the center frequency of the band, and therefore from Eq. (3-6)

$$\Delta T_i = \frac{f_i \sqrt{1/K^2 - 1}}{Q_i |\dot{f}|} . \tag{3-8}$$

For equal time at each of the ith resonances, therefore, the sweep rate must be proportional to frequency and inversely proportional to amplification factor:

$$|\dot{f}| \propto \frac{f_i}{Q_i} .$$

For viscoelastic damping of a specimen of uniform material, Q is constant and the sweep rate is proportional to sweep frequency.

Characteristics of Linear and Logarithmic Sweep Methods. There are two standard sweep methods, linear and logarithmic. In the linear sweep the time rate of change of frequency is constant. This constant h is called the linear sweep rate with units of Hz/sec. In the logarithmic sweep the excitation frequency is varied at a rate proportional to itself. Hence,

$$|\dot{f}| = \frac{f\beta \ln 2}{60} ,$$

where β is the logarithmic sweep rate in octaves/min.

Substitution of these terms for $|\dot{f}|$ into Eqs. (3-6) and (3-8) provides the following relationships for the number of cycles and time spent at each resonance:

$$N_{\text{Linear}} = \frac{f_n^2 \sqrt{\dfrac{1}{K^2} - 1}}{Qh} , \tag{3-9}$$

$$T_{\text{Linear}} = \frac{f_n \sqrt{\dfrac{1}{K^2} - 1}}{Qh} , \tag{3-10}$$

$$N_{\text{Logarithmic}} = \frac{60 f_n \sqrt{\frac{1}{K^2} - 1}}{Q \beta \ln 2}, \tag{3-11}$$

$$T_{\text{Logarithmic}} = \frac{60 \sqrt{\frac{1}{K^2} - 1}}{Q \beta \ln 2}. \tag{3-12}$$

The time required to sweep between a lower and upper frequency limit in a linear sweep in seconds, is

$$T_s = \frac{f_2 - f_1}{h} \,. \tag{3-13}$$

The time required to sweep between two frequencies in a logarithmic sweep, in seconds, is

$$T_s = \frac{60}{\beta \ln 2} \ln \left(\frac{f_2}{f_1}\right). \tag{3-14}$$

Diagnostic Uses of Sweep Tests. A common use of the sinusoidal sweep is the determination of test item dynamic properties and the effect of excitation frequency upon performance characteristics. Resonant frequencies can be determined by monitoring the responses of the item as it is excited by sinusoidal acceleration with slowly varying frequencies. Functional performance of operating equipment can be monitored during the sweep to determine critical frequencies where performance is degraded. It is important in these tests that the sweep rate be slow enough to approximate steady state conditions; as discussed earlier, the amplitude of response and the frequency of peak amplification are dependent on sweep rate.

Transmissibilities and peak amplification factors are properties which can also be determined in a sweep test. (The more general case of modal testing, where detailed dynamic properties such as mode shapes are determined, is discussed on page 68.) The transmissibility is defined as the ratio of a steady state response parameter to a steady state excitation parameter, such as the acceleration response of a part of a specimen divided by the acceleration excitation of the specimen. Conversely, the peak amplification factor Q is a measure of the damping in a particular mode of vibration and is related to the sharpness of the resonant peaks in the transmissibility curves. The value of Q cannot always be determined from the peak value of the transmissibility curve as it is often

incorrectly done. (In certain special cases where the test item may be considered an SDF system the Q may be equal to the peak transmissibility.)

The following example illustrates the difference between Q and transmissibility. Consider a structurally damped two-degree-of-freedom system as shown in Fig. 3-8. The structural coefficient for this system is 0.05, a value equivalent to a Q of 20. The motion transmissibilities between the masses and the foundation are shown in Fig. 3-9. The peak values of the transmissibilities in the two natural modes vary depending on coordinate even though the damping in each mode is equivalent to a Q of 20. The Q's can be determined, however, from the relationship between the half-power bandwidth, natural frequency, and Q from Eq. (3-3);

$$Q = \frac{f_n}{B} .$$

The accuracy of this computation for a particular mode depends not only on instrumentation accuracy and curve resolution, but also on the participation of modes of other natural frequencies. If there are two close natural frequencies, each natural mode may significantly participate in the response at both frequencies. In other words there will be interference and the shape of the resonance curve will include the response motion of more than one mode; thus the computation of the half-power bandwidth will be in error. This is illustrated in Fig. 3-10, a hypothetical transmissibility curve showing two resonances close in frequency. The dotted lines indicate the shape the resonance curve would have if there had been no interference. The error in bandwidth is noted by the difference in widths measured between the solid and the dotted lines. There will always be some interference in the resonance peak from participation of other modes, regardless of the separation in natural frequencies. However, the interference will have less effect on high resonant peaks than it will for low resonant

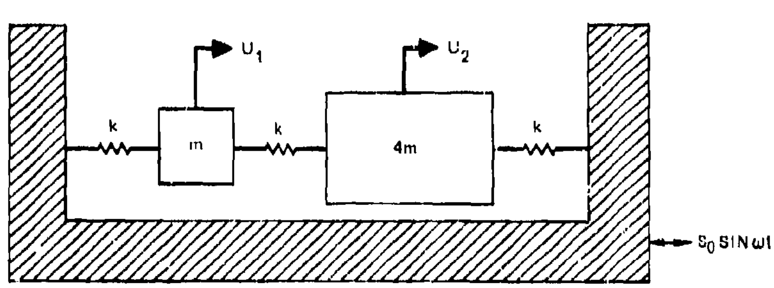

Fig. 3-8. Two-degree-of-freedom model.

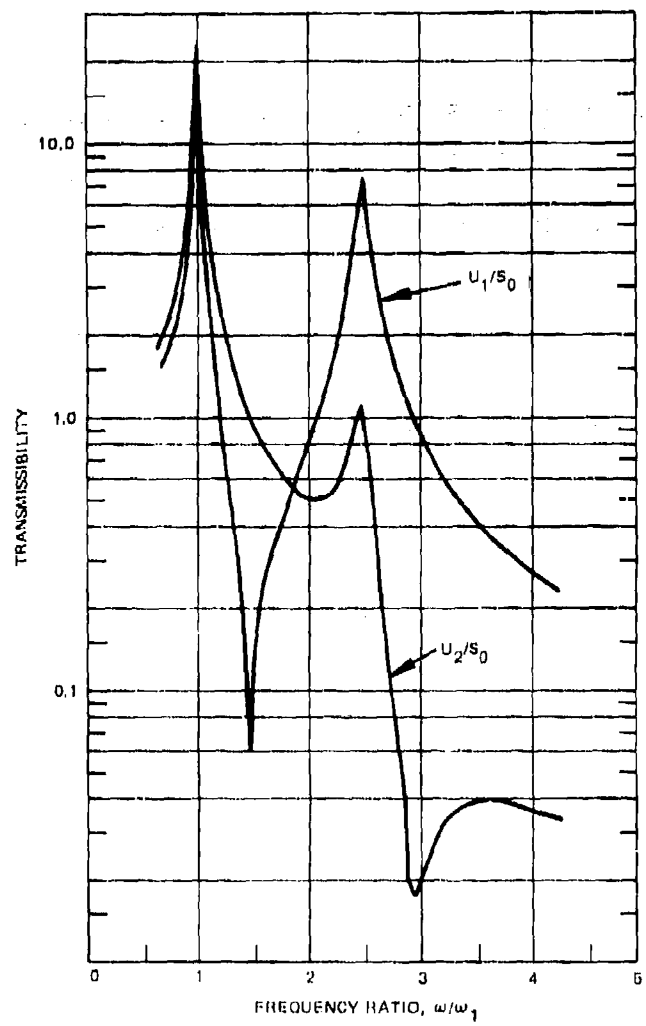

Fig. 3-9. Transmissibilities of two-degree-of-freedom system.

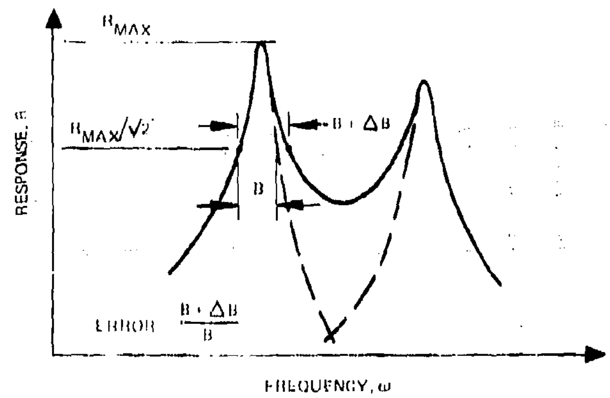

Fig. 3-10. Two close resonances illustrating error in determining peak amplification from measurement of half-power bandwidth.

peaks and, therefore, when there is a choice of transmissibilities from various locations it is best to measure the bandwidth of the highest peak describing the resonance.

Resonance Dwell

In the resonance dwell the test item is excited with constant sinusoidal motion at a series of fixed frequencies corresponding to the resonant frequencies of the test item. The test is designed to simulate the fatigue effects of a vibration environment based on the assumption that fatigue damage is primarily the result of resonant response. This assumption may be warranted, but the test is difficult to relate to service conditions which have random loading histories. The difficulty is in the determination of test conditions, level, and duration that will simulate the service fatigue effects. It is, essentially, a problem of determining an equivalence between sinusoidal and random vibration. Mathematical studies of this equivalence, which are considered in a later section, and experimental studies [39-41] show a wide variation in results, suggesting that an equivalence does not exist except for a few special cases.

The specification for a dwell test should include (1) a definition of the resonant frequencies or a specification of how they are to be determined, (2) the level of excitation, and (3) the duration of each dwell.

The number of frequencies tested is usually less than the number of resonant frequencies the item possesses within the frequency range of the test. This is allowed to enable a minimum duration for each resonance tested within a maximum total test time. The determination of these test frequencies is the

critical factor in dwell testing. For an adequate test the frequencies selected must correspond to the resonances where fatigue failure is most probable in the service environment. To accomplish this the item must receive prior testing and analysis to determine the significant resonances. The significance of the resonances can only be determined by a knowledge of the stresses that occur in each resonance. Many military environmental test specifications suggest that the significance of a resonance can be determined by the level of the transmissibilities measured in a resonance search. They recommend that resonances with peak transmissibilities greater than two be chosen for dwell frequencies. However, since the peak levels of the transmissibility data are dependent on the location of the accelerometers, this method can result in the selection of insignificant resonances and, more important, the significant resonances may go undetected. For example, consider the transmissibilities of the two-degree-of-freedom system shown in Fig. 3-8. An accelerometer placed on the larger mass would measure a peak transmissibility of less than two at the second resonant frequency of the system (see Fig. 3-9). This resonance, which happens to produce high stresses in the springs between the masses, would not be chosen for resonance dwell according to the above criteria.

Modal Testing

Modal tests, commonly referred to in the literature as resonance testing, are carried out to determine experimentally the dynamic parameters of a test item. These measurable parameters are (1) natural frequencies, (2) principal mode shapes, and (3) nondimensional damping factors. A simplified type of modal test is the resonance search test which is carried out to determine resonant frequencies of the test item prior to the resonance dwell test. This section discusses methods for determining mode shapes and damping coefficients which require more complicated test procedures, instrumentation, and data reduction and evaluation. The theory of resonance testing is well documented in papers by Kennedy and Pancu [42] and Bishop and Gladwell [43], which are summarized in the following discussion.

The foundation of normal mode theory is that a structure, when vibrating freely or when forced, has a total motion which is a sum of the motions of individual normal modes. The properties of normal modes are

1. Each normal mode responds to an applied force as a single-degree-of-freedom system i.e., there is no coupling between normal modes.

2. In a normal mode, each point of the system oscillates about its equilibrium position along a certain line in space, fixed relative to the equilibrium position and straight when the oscillations are small enough so that all angles are equal to their sines.

3. In the case of simple harmonic vibration in a normal mode, all points move either exactly in or exactly out of phase with each other. That is, all points reach maximum departures from their equilibrium positions at the same instants.

4. The shape of each normal mode is fixed for a given system and is independent of the magnitude, frequency, or location and direction in space of the applied external forces or of the deflections in other normal modes present. That is, in any given normal mode, the ratios of the deflections at all the points of a structure to the deflection at an arbitrary reference point are always constant, and the directions of these deflections are fixed in space. These relative magnitudes and directions in space are characteristic of the normal mode, and their specification for every point in the structure will be referred to as the description of the "shape" of that mode.

The properties of normal modes hold rigorously only for proportional damping, where the damping matrix is proportional to the stiffness or inertia matrices, or for zero damping. Damping, which is always present in actual systems, may or may not be proportional. For the mathematics to be tractable, however, the assumption of proportional damping is required. Because the damping forces are small in actual systems the errors in this assumption are not great. Some experimental methods described by Bishop and Gladwell do not require the assumption of proportional damping, and the reader is referred to their paper for descriptions of these methods.

Because the shape of each mode is fixed it is possible to describe the motion of each mode by a single coordinate q_n which is called the normal coordinate. The physical coordinates of the system are related to the normal coordinates by the linear transformation

$$U_i(t) = \sum_{n=1}^{N} \phi_{in} q_n(t),$$

where ϕ_{in} represents the amplitude of the ith coordinate when the system is vibrating in a single mode of frequency ω_n. The array of N elements ϕ_{in} is the mode shape for the nth mode.

The equation of motion for the coordinate of a normal mode has the same form as the equation for an SDF system

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = F(t),$$

where ζ_n is the damping factor for the nth mode and ω_n is the natural frequency.

The problem in resonance testing is to determine the mode shapes ϕ_{in}, the damping factors ζ_n, and the natural frequencies ω_n from the measurable quantity $U_i(t)$, the displacements of the system. Bishop and Gladwell discuss three separate techniques for resonance testing: (1) the peak-amplitude method, (2) the Kennedy and Pancu method, and (3) methods involving pure-mode excitation.

In the peak-amplitude method the structure is excited harmonically and the amplitudes at various points are measured. Total amplitude is plotted against excitation frequency for the various locations. The natural frequencies are identified as the values of ω at which the peaks are attained. The damping factors are determined by measuring the sharpness of the peak about the natural frequencies. This measurement requires the assumption that the peaks are solely the result of responses in a single normal mode; as discussed on page 65 and illustrated in Fig. 3-10. The shapes of the principal modes are calculated from the ratios of the amplitudes at various points when the structure is being excited at a natural frequency. Accurate mode shapes are difficult to obtain and there are several reasons for this. The primary reason is that the response amplitudes at a natural frequency of a harmonically excited structure are composed of components of several natural modes. Whereas the damping factor is calculated from one peak about which there is some uncertainty, the mode shape is calculated from the ratios of a number of peaks about each of which there is uncertainty. This means that if an error is made in estimating the contribution to any peak from the resonant mode, the error in the mode shape is likely to be many times greater than that in the damping factor. There is also a practical difficulty in the determination of mode shapes due to problems of maintaining a constant excitation force near a natural frequency of a structure.

The method of Kennedy and Pancu differs from the peak-amplitude method in its approach to the measurement of the damping factors and mode shapes. Instead of measuring just the amplitude of the vibration, the amplitude and phase are measured. The major cause of inaccuracy in the estimation of the damping factors and mode shapes in the peak-amplitude method is the presence near the resonant frequencies of unknown amounts of vibration in the off-resonant modes. Kennedy and Pancu make use of the phase relationship properties of normal modes to extract from the total vibration the vibration of a single normal mode. Their method involves considerably more data reduction and evaluation and more care in the control of the excitation forces than the peak-amplitude method. The accuracy of the results however, is considerably greater than those obtained in the peak-amplitude method.

The third technique for obtaining the dynamic properties of a structure is termed pure-mode excitation. The difficulty in interpreting response amplitude data when the vibration is composed of several modes can be eliminated if the structure can be made to vibrate in a single normal mode at a time. To get the system to vibrate in a single mode requires a forcing function which is decoupled from all other modes except the one in question. This can be accomplished only if the distribution of the force has the same shape as the mode being determined (i.e., the forcing function must be orthogonal to all other modes). Since it is precisely this shape which is being determined, the experimental difficulty is apparent. Systematic iteration procedures involving multiple excitation are required. Two such procedures, reviewed by Bishop and Gladwell, are (1) the method of Lewis and Wrisley, and (2) the methods of Traill-Nash and Asher.

Both of these methods require multiple excitation and considerable data processing and evaluation. The reader is referred to the reference material for descriptions of these methods.

3.3 Random Test Methods

The advisability of using random vibration to simulate field environments was first suggested in the 1950's [44]. Since that time its use has become increasingly popular, so that today most vibration laboratories have capabilities for producing broadband random motion. The increasing use of random vibration was brought about by the fact that most of the vibration encountered by military equipment is random in time rather than periodic and has a frequency spectrum which is continuous rather than discrete, and by the realization that, in general, it is not possible to simulate one type of vibration by another, i.e., the simulation of random with fixed or varying frequency sinusoids.

In random testing the excitation waveform has a normal or Gaussian instantaneous amplitude distribution. The test amplitude and frequency are described by the acceleration spectral density vs frequency, or simply the spectral density curve. The standard random vibration test is broadband, usually 20 to 2000 Hz, with relatively constant spectral density. In cases where the environment is more completely defined, the spectral density may vary significantly with frequency.

Characteristics of Random Vibration

A random time function consists of a continuous distribution of sine waves at all frequencies, the amplitudes and phase angles of which vary in an unpredictable (random) manner as a function of time.

Such a function may be visualized as follows: Consider an oscillator which generates a sinusoid

$$\xi(t) = s(t) \sin [\omega t + \phi(t)] .$$

The amplitude $s(t)$ and the phase angle $\phi(t)$ both vary with time in a random fashion. If the outputs of a number of such oscillators with different frequencies ω are added together, something close to a random function as defined above is obtained. In the limit as the number of oscillators approaches infinity so that there is essentially zero frequency difference between adjoining oscillators, then a random function with a continuous spectrum is formed.

The mean square value of the random function is

$$\overline{\xi^2} = \frac{1}{T} \int_0^T [\xi(t)]^2 \, dt = \sigma^2 . \tag{3-15}$$

The rms value σ is the square root of the mean square (m.s.) value.

Two other statistical parameters used in describing random functions are the probability density function and probability distribution function.

The probability density function $p(\xi/\sigma)$, illustrated in Fig. 3-11, defined the probability (or fraction of time, on the average) that the magnitude of the quantity $\xi(t)$ will lie between two values. It is customary to normalize the curve by plotting the magnitude divided by the rms value σ as the abscissa. Then the probability that the magnitude lies between ξ/σ and $(\xi + d\xi)/\sigma$ is equal to $p(\xi/\sigma)$ $d(\xi/\sigma)$, i.e., the shaded area shown in Fig. 3-11. Since it is certain, with probability 1.0, that the function $\xi(t)$ lies between plus and minus infinity, the area under the entire curve is unity.

The probability distribution function (or cumulative distribution function) $P(\xi/\sigma \geqslant)$,* defines the probability that the magnitude of ξ/σ will exceed a certain value

A comparison of the identifying characteristics of the probability densities and distributions for the instantaneous values of a sinusoidal function and the

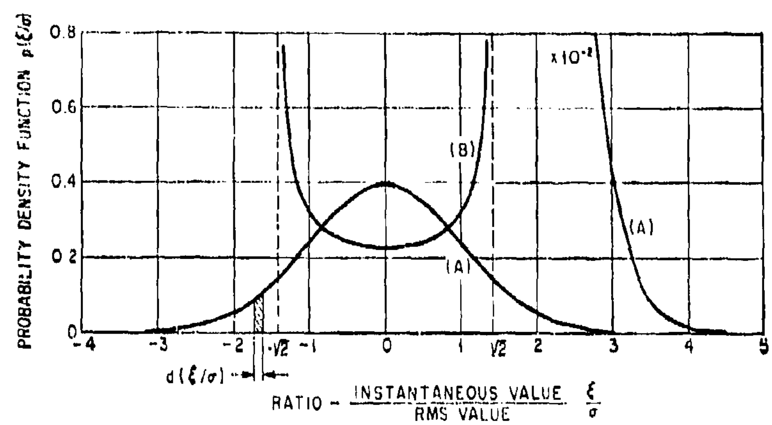

Fig. 3-11. Normalized probability density functions—(A) Gaussian or normal distribution and (B) Distribution of instantaneous values of a sine wave. Curve (A) marked $\times 10^{-2}$ indicates hundredfold expansion of the ordinate scale. From *Shock and Vibration Handbook*, vol. 2, Fig. 22.7, p. 22-7; copyright 1961 by McGraw-Hill Book Company, Inc. Used by permission.

*The probability that ξ/σ is "greater than" is written $P(\xi/\sigma \geqslant)$; conversely, the probability that ξ/σ is "less than" is written $P(\xi/\sigma \leqslant)$. $P(\xi/\sigma \leqslant) = 1 - P(\xi/\sigma \geqslant)$.

particular case of Gaussian random noise is shown in Figs. 3-11, 3-12, and 3-13. Gaussian noise is a random function whose instantaneous value is defined by the Gaussian or normal probability density function given by

$$p(\xi/\sigma) = \frac{1}{\sqrt{2\pi}} e^{-(\xi^2/2\sigma^2)}, \tag{3-16}$$

where σ is the rms value, and is shown by curve (A) of Fig. 3-11. The probability density function of the instantaneous value of a sinusoid is shown by curve (B) of Fig. 3-11 and is defined by

$$p(\xi/\sigma) = \frac{1}{\pi\sqrt{2 - (\xi/\sigma)^2}} . \tag{3-17}$$

The density functions of Eqs. (3-16) and (3-17) are symmetrical about a mean assumed to be zero; then the probability that ξ exceeds a given absolute value (or magnitude) $|\xi|$ is twice the probability that it exceeds the same absolute

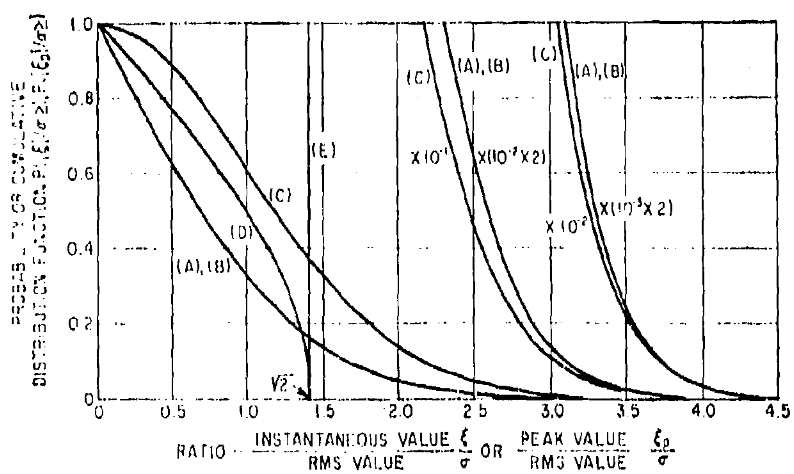

Fig. 3-12. Probability distribution functions – multiply ordinate scale by factors marked adjacent to curves for large abscissa values — (A) Instantaneous values of broadband and narrowband random variation – Gaussian distribution, (B) Peaks of broadband random vibration, (C) Peaks of narrowband random vibration – Rayleigh distribution, (D) Instantaneous values of a sine wave, and (E) Peak values of a sine wave. From *Shock and Vibration Handbook*, vol. 2, Fig. 22.9, p. 22-9; copyright 1961 by McGraw-Hill Book Company, Inc. Used by permission.

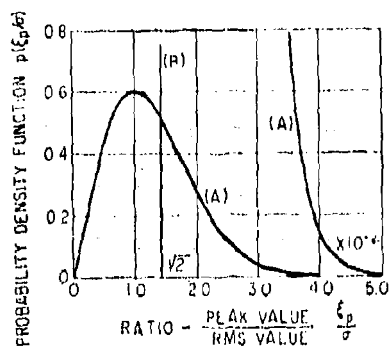

Fig. 3-13. Normalized probability density functions - curve (A) marked $\times 10^{-2}$ indicates hundredfold expansion of ordinate scale (A) Rayleigh distribution for peaks of narrowband Gaussian vibration, and (B) Distribution for peaks of sine wave delta function at $\xi_p/\sigma = \sqrt{2}$. From *Shock and Vibration Handbook*, vol. 2, Fig. 22.8, p. 22-8; copyright 1961 by McGraw-Hill Book Company, Inc. Used by permission.

value in either the positive or negative sense. Therefore, it is convenient to plot the probability distribution function in terms of the absolute value of ξ, i.e., $P(\xi/\sigma \geqslant)$, as shown in Fig. 3-12.

The probability distribution functions for the Gaussian and sinusoidal functions are obtained by integrating Eqs. (3-16) and (3-17).

$$\text{Gaussian:} \qquad P(|\xi|/\sigma \geqslant) = \frac{2}{\sqrt{2\pi}} \int_{\xi/\sigma}^{\infty} e^{-(\xi^2/2\sigma^2)}\, d(\xi/\sigma), \qquad (3\text{-}18)$$

$$\text{Sinusoidal:} \qquad P(|\xi|/\sigma \geqslant) = \frac{2}{\pi} \cos^{-1} \frac{\xi}{\sigma\sqrt{2}}. \qquad (3\text{-}19)$$

The relations of these equations are plotted as curves (A) and (B) in Fig. 3-12.

When the peak values or maxima of a function are considered, the two statistical functions differ from those found for the instantaneous values. For a sine wave, all maxima are of equal magnitude and the probability density function $P(\xi/\sigma)$ becomes a Dirac delta function, as shown by curve (B) of Fig. 3-13. For broadband Gaussian noise, i.e., noise with nonzero amplitude over a frequency band which is not small compared to the average or center frequency of the band, the distribution of peak values is also normal, as shown by curve (B) of Fig. 3-12. However, for narrowband Gaussian noise, i.e., noise with negligible amplitude except in a frequency bandwidth which is small compared to the

center frequency, the distribution of peak values tends toward the Rayleigh distribution. The probability density and distribution functions for the Rayleigh distribution are defined below.

$$\text{Probability Density:} \quad p(|\xi_p|/\sigma) = (|\xi_p|/\sigma)\, e^{-\xi_p^2/2\sigma^2} \,. \tag{3-20}$$

$$\text{Probability Distribution:} \quad P(|\xi_p|/\sigma) = e^{-\xi_p^2/2\sigma^2} \,. \tag{3-21}$$

The relations given by these equations are shown graphically by curve (A) of Fig. 3-13 and curve (C) of Fig. 3-12, respectively. These figures show that the probability density and probability distribution curves for sinusoidal and random functions differ considerably, thus providing identifying characteristics for each type of function. When the time history $\xi(t)$ consists of sinusoidal and random functions, the shape of the probability density and distribution curves depends on the relative magnitude of each type of function.

Power spectral density* is defined as the limiting value of the mean square response $\overline{[\xi']^2}$ of an ideal bandpass filter† to $\xi(t)$, divided by the bandwidth B of the filter, as the bandwidth of the filter approaches zero.

An alternative definition is as follows. If the function $\xi(t)$ is passed through an ideal lowpass filter‡ with cutoff frequency f_c, the mean square response of the filter $\overline{[\xi']^2}$ will increase or decrease as f_c is increased or decreased, i.e., more or less of the function will be passed by the filter (assuming f_c is varied in a frequency range where the power spectral density is nonzero). The power spectral density $W(f)$ is the rate of change of $\overline{[\xi']^2}$ with respect to f_c, i.e.,

$$W(f_c) = \frac{d}{df_c}\left\{\overline{[\xi']^2}\right\} \tag{3-22}$$

Fig. 3-14 illustrates a plot of power spectral density as a function of frequency obtained, for example, from spectral analysis of a random function. The mean

*Power spectral density is defined by Eq. (3-22). It is a generic term used regardless of the physical quantity represented by the time history. However, it is preferable to indicate the physical quantity involved. For example, use the term *mean square acceleration density* when the time history of acceleration is to be described.

†An ideal bandpass filter has a transmission characteristic which is rectangular in shape so that all frequency components within the filter bandwidth are passed with unity gain and zero phase distortion, while frequency components outside the bandwidth are completely removed. If the transmission characteristic is H instead of unity in the bandwidth B, the spectral density is obtained by dividing the mean square response by $B \times H^2$ instead of B.

‡An ideal lowpass filter is an ideal bandpass filter having a lower cutoff frequency of zero.

square value or variance of the frequency content of $\xi(t)$ between the frequencies f_a and f_b is equal to the shaded area of Fig. 3-14; i.e.,

$$\sigma^2 (f_a \leq f \leq f_b) = \int_{f_a}^{f_b} W(f)\, df. \tag{3-23}$$

When a sample of time history of finite duration is employed to compute the power spectral density of a random function, it is assumed that (1) the function is ergodic, i.e., that averaging one time history with respect to time yields the same results as averaging over an ensemble of time histories at a given instant of time, and (2) that the function is stationary, i.e., that the power spectral density is independent of the sample of the time history chosen. Further, the averaging time or sample duration must be long enough to yield a statistically significant value. Thus, the mean square value obtained should not vary appreciably with a change in averaging time. The time over which a vibration record may be considered a stationary process and the need for a sufficiently long averaging time often are conflicting requirements.

Response of Mechanical Systems to Random Vibration

The following material is confined to steady state responses of idealized linear mechanical systems. Further, only random functions which have a Gaussian or normal distribution will be discussed. Almost all analyses to date have been made on this assumption, since (1) the analysis then becomes tractable, and (2) most physical processes including laboratory tests follow the Gaussian distribution.

Computation of the response of this idealized system can be briefly described as follows. The excitation forces (or motions) acting on the system are assumed

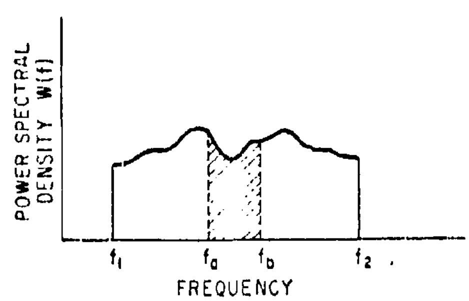

Fig. 3-14. Typical power spectral density plot of broadband random function. Mean square value of frequency content of function between f_a and f_b is equal to shaded area. From *Shock and Vibration Handbook*, vol. 2, Fig. 22.16, p. 22-16; copyright 1961 by McGraw-Hill Book Company, Inc. Used by permission.

to be known in terms of a power spectral density $f(\omega)$. If the response of the system to harmonic excitation as a function of excitation frequency is computed, i.e., frequency response function $H(\omega)$, the response to the random function is obtained as follows [45]:

$$g(\omega) = f(\omega)\,|H(\omega)|^2\,, \tag{3-24}$$

where $g(\omega)$ is the response spectral density.

The mean square response $\overline{R^2}$ in any desired frequency band is equal to the integral of the response spectral density between the frequency limits of the band, i.e., the area under the response spectral density curve;

$$\overline{R^2} = \int_0^\infty g(\omega)\,d\omega = \int_0^\infty f(\omega)\,|H(\omega)|^2\,d\omega\,. \tag{3-25}$$

It is worthwhile to spend some time examining the characteristics of the response. This is easily carried out using the SDF system shown in Fig. 3-1.

The base acceleration of this system $\ddot{S}(t)$ is defined by a uniform acceleration spectral density W in units of g^2/Hz. The absolute value of frequency response function $|H(\omega)|$, which relates the relative acceleration response $\ddot{Y}(t)$ to the input acceleration $\ddot{S}(t)$, is equal to the amplification factor H described in Eq. 3-2. Substitution of these values into Eq. (3-24) results in

$$g(\omega) = WH^2 = \frac{W}{\left[1-(\omega/\omega_n)^2\right]^2 + (2\zeta\,\omega/\omega_n)^2}\,.$$

where $g(\omega)$ is the response acceleration spectral density for a white noise base excitation of level W. This function is illustrated in Fig. 3-15. Note that the response is concentrated in a narrow frequency band centered about the natural frequency ω_n. The mean square response of $\ddot{Y}(t)$ can be determined from Eq. (3-25) as follows:

$$\overline{\ddot{Y}^2} = \int_0^\infty g(\omega)\,d\omega = \int_0^\infty \frac{W\,d\omega}{\left[1-(\omega/\omega_n)^2\right]^2 + (2\zeta\,\omega/\omega_n)^2}\,. \tag{3-26}$$

By assuming $\zeta << 1$, integration can be performed resulting in

$$\overline{\ddot{Y}^2} = \frac{\pi Q f_n W}{2}\,, \tag{3-27}$$

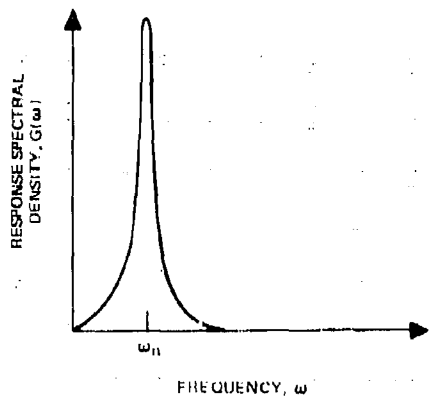

Fig. 3-15. Response spectral density of an SDF system excited by white noise.

where $\overline{\ddot{Y}^2}$ is the mean square relative acceleration in units of g^2 for input spectral density in units of g^2/Hz.

Terms for the mean square relative velocity and displacement can also be determined from Eq. (3-25) by substitution of the frequency response function relating the relative velocity and displacement to the base acceleration. These functions are

a. Relative velocity

$$H(\omega) = \frac{i\,\omega}{\omega^2 - \omega_n^2 - 2 i \zeta \omega_n \omega} ,$$

$$\overline{\dot{Y}^2} = \int_0^\infty W\,|H(\omega)|^2\, d\omega = \frac{W\,Q\,(386)^2}{8\,\pi\, f_n} . \qquad (3\text{-}28)$$

where $\overline{\dot{Y}^2}$ is the mean square relative velocity in in.^{2}/sec^2 for a spectral density input in g^2/Hz.

b. Relative displacement

$$H(\omega) = \frac{1}{\omega^2 - \omega_n^2 - 2 i \zeta \omega_n \omega} ,$$

$$\overline{Y^2} = \frac{W\,Q\,(386)^2}{4(2\pi f_n)^3}\ , \tag{3-29}$$

where $\overline{Y^2}$ is the mean square relative displacement in square inches for a spectral density input in g^2/Hz.

Since the response spectral density is concentrated in the region of the natural frequency, the time history of the response is narrowband random vibration and is similar to a sine wave at that frequency whose amplitude fluctuates randomly at a rate roughly equal to the frequency band of the resonance. The fluctuation is associated with the phenomenon of beats, where the addition of two harmonic waves of approximately equal frequency results in a wave with a frequency equal to the mean of the two frequencies and an amplitude envelope that fluctuates at a frequency equal to the difference of the two frequencies. The narrowband random time history is composed of a large number of waves of nearly equal frequencies with randomly distributed phases and amplitudes and thus oscillates at a frequency f_n, with a random amplitude envelope that fluctuates at a frequency approximately equal to the half-power bandwidth f_n/Q. This type of waveform is illustrated in Fig. 3-16.

For linear systems the instantaneous amplitudes of the narrowband response have a Gaussian distribution while the distribution of peak amplitudes follows the Rayleigh distribution (see Figs. 3-11 and 3-13).

3.4 Equivalence in Vibration Testing

Instead of simulating a service environment with a test method that resembles the basic characteristics of that environment, it is sometimes necessary to substitute a method which is different in character but equivalent in its effect on the test specimen. The substitution may be required because of equipment, cost, schedule, or technical limitations. For example, the lack of equalization equipment may require the use of a sinusoidal test instead of a random test; time limitations may require that a test duration be decreased with a corresponding increase in excitation level; the inability to detect or decide upon significant resonances may dictate the use of a sinusoidal sweep rather than a sinusoidal dwell.

When a test is equivalent to another the two tests should produce equal effects on the test specimen. That is, the damage caused by each of the two methods should be equal. This is usually not possible to accomplish if more than one effect is to be simultaneously simulated. For example, it may be possible to simulate the fatigue damage caused by a random environment with a sinusoidal sweep, but it is unlikely that the same sinusoidal test can simulate both the

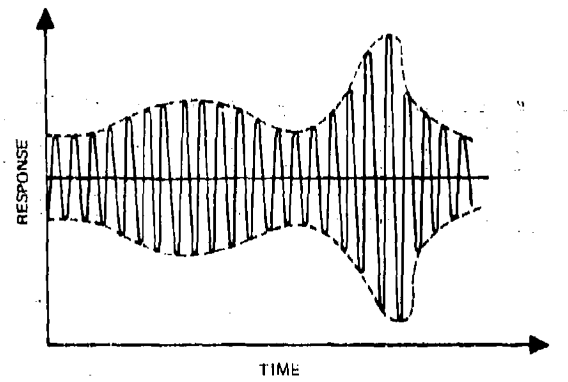

Fig. 3-16. Narrowband random time history.

fatigue and the effects the random environment has on functional performance of the test item. Therefore, equivalence between two test methods implies the equivalence of the single most damaging effect of the environment.

The following sections contain discussions of the equivalence between the three standard test methods for fatigue, resonant response, energy dissipation, and functional performance. In addition, a discussion is included of the relationship between test time and test level which provides an equivalence between two tests of the same type but at different levels and durations.

Structural Fatigue

A large number of vibration failures are considered to be the result of structural fatigue damage. It is appropriate, therefore, that equivalences be derived between the three standard test methods sinusoidal dwell and sweep, and broadband random that are based on producing equal fatigue damage [46].

There has been much written on fatigue equivalence, with widely varying results. Considering that the prediction of fatigue life is, even for well-defined structural materials, a probabalistic procedure, the different results are not surprising and are in fact expected. The exact nature of fatigue failures is not fully understood, and to a large extent fatigue relationships are based on the study of experimental results where there is a significant amount of scatter.

The fatigue properties of a material are determined by subjecting specimens of that material to alternating stress until failure. Tests are repeated until enough data are provided to define a curve of stress vs cycles to failure, typically called the endurance or σ-N curve. It shall be assumed that this curve can be described by the following equation:

$$N\sigma^b = c, \qquad (3\text{-}30)$$

where σ is the stress amplitude, N is the number of cycles to failure, and b and c are positive material constants. The wide scatter in fatigue data would suggest that b and c are random variables. However, in the following sections they are assumed constant.

For constant cyclic loading, such as sinusoidal dwell, Eq. (3-30) can be used directly to predict fatigue life. However, for random and sinusoidal sweep, where the loading is variable, a cumulative damage theory is required to predict fatigue life. The most widely used theory of cumulative damage is referred to as Miner's Rule [47-49] and is expressed as

$$D = \sum \frac{n_i}{N_i} \,, \tag{3-31}$$

where n_i is the number of cycles at stress σ_i and N_i is the number of cycles to failure at stress σ_i (from Eq. 3-30). D is the measure of accumulated damage and according to Miner, failure is predicted for $D \geqslant 1$. Experiments have shown that the value D has a rather wide scatter depending on the sequential loading history. Generally, investigators found that if the stress level was changed from a high to a lower value the summation of cycle ratios was less than unity. Conversely, a sum greater than one was frequently observed when the stress level was increased during a test.

Very probably some of the apparent inconsistency in the value of D achieved in a given set of tests is due to inaccuracies in the σ-N curves used to perform the calculations. Recent reviews by Hardrath [50], Richards and Mead [51], and Grover [52], reveal that no trend is evident that any estimation procedure is superior to the linear method.

In the following sections where the equations for fatigue damage due to sinusoidal sweep and random vibration are developed, it is assumed that the majority of damage is the result of resonant response. As in earlier sections the equations of motion of an SDF system with viscous damping are used in the development of the results.

Fatigue Damage from Resonance Dwell. The resonance dwell test is defined as constant sinusoidal excitation at a specimen resonant frequency. The stress σ produced at resonance is proportional to the excitation level $\ddot{S}_0$ and the amplification factor Q;

$$\sigma = K_s Q\ddot{S}_0 \,,$$

with K_s the proportionality constant. The number of cycles to failure at this stress level can be determined from Eq. (3-30) by substitution;

$$N = c\sigma^{-b} = c(K_s Q\ddot{S}_0)^{-b} .$$

Thus, the time to failure T_c at the resonant frequency f_n is

$$T_c = \frac{c}{f_n}(K_s Q\ddot{S}_0)^{-b}. \tag{3-32}$$

Fatigue Damage due to Sinusoidal Sweep. In a sinusoidal sweep significantly high stress cycles occur in frequency bands centered about the resonant frequencies of the test specimen. As the varying excitation frequency passes through a resonance the stress levels increase and then decrease with the maximum level occurring at the natural frequency. Each cycle of varying stress must be accounted for to evaluate the total fatigue damage resulting from sweep excitation of the resonance. The stress at any frequency can be found from

$$\sigma = K_s\,|H(\omega)|\,\ddot{S}_0,$$

where $|H(\omega)|$ is a transfer function between response motion and input motion.

Substituting in Eq. (3-30) results in

$$N = c\left(K_s|H(\omega)|\ddot{S}_0\right)^{-b}.$$

This relationship is then used in Miner's Rule (Eq. 3-31) which is expressed in integral form

$$D_s = \int \frac{(K_s\,|H(\omega)|\,\ddot{S}_0)^b}{c}\,dn,$$

where D_s is the damage coefficient for sinusoidal sweep.

With a sweep rate slow enough to produce 99 percent steady state response, the amplification term $|H(\omega)|$ is approximated in the bandwidth of the resonance by the steady state amplification factor H for a base-excited SDF system (Eq. 3-2),

$$H(\omega) = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}},$$

where r is the ratio of excitation frequency to natural frequency f/fn. This excitation frequency is related to the sweep rate $|\dot{f}|$ and time of sweep by

$$f = |\dot{f}|t.$$

Since the majority of damage occurs from stress cycles within a narrow bandwidth centered about the resonant frequency,* the sweep rate can be considered constant and the above relationship is expressed as

$$df = h\,dt\ ,$$

where h is the linear sweep rate in Hz/sec.

The number of cycles in an infinitesimal amount of sweep time is

$$dn = f\,dt\,.$$

Thus,

$$dn = \frac{f_n^2\, r\,dr}{h}\ .$$

Substitution of these relationships into the equation for D_s results in

$$D_s = \int \frac{\left(K_s\,|H(\omega)|\,\ddot{S}_0\right)^b dn}{c} = \int \frac{\left(K_s\,|H(\omega)|\,\ddot{S}_0\right)^b f_n^2\, r\,dr}{ch}\ ,$$

$$D_s = \frac{\left(f_n^2\ \ K_s\,\ddot{S}_0\right)^b}{ch} \int_{r_1}^{r_2} \frac{r\,dr}{\left[(1-r^2)^2+(2\,\zeta\,r)^2\right]^{b/2}}\ . \tag{3-33}$$

The limits of integration r_1 and r_2 bracket the half-power bandwidth of the resonance.

The values of the integral in Eq. (3-33) determined by numerical solution are shown in Fig. 3-17. A simple function can be fitted to these curves to provide an approximate description of the integral

$$\int_{r_1}^{r_2} \frac{r\,dr}{\left[(1-r^2)^2+(2\,\zeta\,r)^2\right]^{b/2}} = \frac{\pi}{2}\ Q^{(b-1)}\ b^{-1/\sqrt{\pi}}\ ,$$

which, by substituting in Eq. (3-33), results in

*Damage actually results from all stress cycles, but the error in assuming that all of the damage occurs within the half-power bandwidth f_n/Q is less than 3 percent.

$$D_s = \frac{\pi f_n^2 (K_s \ddot{S}_0 Q)^b \, b^{-1/\sqrt{\pi}}}{2 c h Q}$$

which is valid for $b < 30$ and $\zeta < 0.1$.

Since the time to sweep the half-power bandwidth is f_n/Qh, the above equation can be expressed in terms of time to failure by

$$T_s = \frac{2c \, b^{1/\sqrt{\pi}} D_s}{\pi f_n (K_s \ddot{S}_0 Q)^b}. \qquad (3\text{-}34)$$

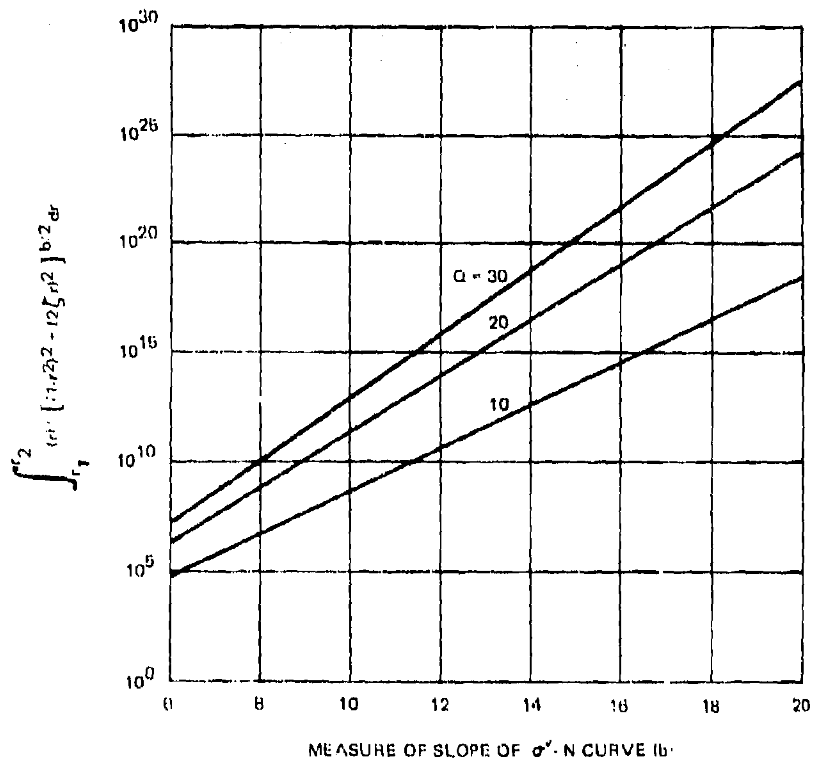

Fig. 3-17. Values of the integral in Eq. (3-33) vs b for various values of amplification factor.

Equations (3-34) and (3-32) are combined to show the sweep duration necessary for equivalent fatigue between a sinusoidal sweep and a resonance dwell;

$$T_s = \frac{2\, D_s' \, b^{\,1/\sqrt{\pi}}}{\pi} \quad T_c' \, . \tag{3-35}$$

The equivalence will vary according to the constant D_s, the damage coefficient for variable loading, and b, the measure of the slope of the σ-N curve.

Endurance tests with numerous different types of stress distribution, both ordered and random, show that the value D_s ranges between 1/5 and 2/3, with a majority of the data near $D_s = 1/2$ [53, 54].

A review of the literature of fatigue and endurance data reveals that b is between 3 and 25. Lunney and Crede [55] and Gertel [56] indicate that a value of 9 is representative of the majority of the materials.

Substitution of these limiting and average values for the constants results in

$$0.24 < T_s/T_c' < 2.85$$

and, on the average, $T_s/T_c = 1.1$.

Fatigue Damage due to Random Loading. The fatigue damage of a specimen subjected to random loading has been examined by Miles [57]. His analysis, which utilizes Miner's cumulative damage hypothesis, results in an equivalent stress which together with the conventional endurance curve can be used to predict the fatigue life of a specimen.

As in the case of sinusoidal sweep excitation, it is necessary to assume that the fatigue damage is the result of resonant response. The stress waveform, therefore, will have the characteristics of narrowband random response as described in Section 3.3 (p. 79) and illustrated in Fig. 3-16. The probability density of the peaks of the stress conforms to a Rayleigh distribution and from Eq. (3-20) can be described by

$$p\left(\frac{|\sigma_p|}{\sqrt{\overline{\sigma}}}\right) = \left(\frac{|\sigma_p|}{\sqrt{\overline{\sigma}}}\right) e^{-(\sigma_p^2/2\overline{\sigma^2})} \, , \tag{3-36}$$

where

σ_p = values of the peak stress,

$\overline{\sigma^2}$ = mean square stress.

From Miner's Rule (Eq. 3-31) the damage from the accumulation of stress cycles at random amplitudes σ_p is

$$D_r = \sum_i \frac{\sigma_{pi}^b n_i}{c},$$

where constants b and c are those of Eq. (3-30).

It is convenient to introduce an equivalent stress S_e and define it as a constant amplitude stress which will produce equal damage as the variable stress σ_p after the same number of total cycles. Thus,

$$D_c = \sum_i \frac{S_e^b n_i}{c}.$$

The variability of the damage coefficients D_c for constant loading and D_r for random loading, for which failure is predicted, are discussed on page 85. Noting that for constant cyclic loading the coefficient is unity, we combine the above relationships to give

$$\sum \frac{S_e^b n_i}{c} = \frac{1}{D_r} \sum \frac{\sigma_{pi}^b n_i}{c},$$

or

$$S_e = \left(\frac{1}{D_r}\right)^{1/b} \left(\frac{\sum_i \sigma_{pi}^b n_i}{\sum_i n_i}\right)^{1/b}$$

With the assumption that the stress wave is oscillating with completely reversed loading at the natural frequency f_n, the probable number of cycles of loading having an amplitude in the range $(\sigma, \sigma + d\sigma)$ is obtained by multiplying the total number of cycles by the Rayleigh distribution density (Eq. 3-36). The above equation therefore reduces to

$$S_e = \left(\frac{1}{D_r}\right)^{1/b} \left[\int_0^\infty \sigma_p^b\, p(\sigma)\, d\sigma \Big/ \int_0^\infty p(\sigma)\, d\sigma\right]^{1/b}.$$

The evaluation of this integral was performed by Miles with the following result:

$$S_e = \left(\frac{1}{D_r}\right)^{1/b} (\pi b)^{1/2b} \left(\frac{b}{e}\right)^{1/2} \sqrt{\overline{\sigma^2}}, \tag{3-37}$$

where $\sqrt{\overline{\sigma^2}}$ is the root mean square stress of the narrowband response.

For a base-excited system subjected to broadband random acceleration, the rms stress is, from Eq. (3-27),

$$\sqrt{\overline{\sigma^2}} = K_s \sqrt{\frac{\pi}{2} f_n Q W},$$

and substituting into Eq. (3-37) results in

$$S_e = \left(\frac{1}{D_r}\right)^{1/b} (\pi b)^{1/2} \left(\frac{b}{e}\right)^{1/2} \left[\frac{\pi}{2} f_n Q W\right]^{1/2} K_s .$$

This relationship can then be used in Eq. (3-30) and rearranged to provide an expression for the time to failure under random loading:

$$T_r = \frac{eD_r}{f_n K_s^b (\pi b)^{1/2} \left(\dfrac{\pi f_n Q W b}{2e}\right)^{b/2}} . \tag{3-38}$$

Random/Sinusoidal Dwell Fatigue Equivalence. Equation (3-38) equated to the time to failure for a sinusoidal dwell (Eq. 3-32) results in an equivalence in spectral density level W and sinusoidal excitation level S_0, based on equal fatigue damage:

$$\ddot{S}_o = \frac{(\pi b)^{1/2b} \left(\dfrac{\pi f_n W b}{2eQ}\right)^{1/2}}{(D_r)^{1/b}} . \tag{3-39}$$

The constants D_r and b have the values

$$1/5 < D_r < 2/3, \text{ and, on the average, } D_r = 1/2$$

$$3 < b < 25, \text{ and, on the average, } b = 9.$$

Substitution of these limiting and average values for the constants results in

$$\ddot{S}_0 = K\left(\frac{f_n W}{Q}\right)^{1/2} ,$$

where

$$2.23 < K < 4.16,$$

and, on the average, $K = 2.6$.

Random/Sinusoidal Sweep Fatigue Equivalence. Equations (3-38) and (3-34) are equated to determine an equivalence in spectral density and sinusoidal sweep acceleration $\ddot{S}_0$, with the condition that the time to sweep the half-power bandwidth is equal to the random test duration;

$$\ddot{S}_0 = \left(\frac{2}{\pi}\right)^{1/b} b^{1/b}\sqrt{\pi}\,(\pi b)^{1/2b}\left(\frac{\pi f n\, W b}{2eQ}\right)^{1/2} , \qquad (3\text{-}40)$$

or

$$\ddot{S}_0 = K\left(\frac{f_n W}{Q}\right)^{1/2} ,$$

where $1.96 < K < 4.4$, and, on the average, $K = 2.66$.

Accelerated Testing. An equivalence which is important in vibration testing is the relationship between vibration and time. For example, electronics in aircraft typically have a five-year life requirement which could consist of thousands of hours of flight vibration of variable intensity. It is not practical to test equipment for these durations. To compress the many hours of field environment into an equivalent test requires a relationship between time and vibration level

If it is assumed that stress is proportional to excitation level, it is possible to apply Miner's hypothesis for cumulative fatigue damage and thereby relate number of cycles or time at one excitation to an equivalent excitation level for a fewer number of cycles. All that is required for this equivalence relationship is the slope of the σ-N curve.

However, experimental evidence reveals that the assumption of a linear relationship between stress and excitation is an invalid assumption. Stress level seems to increase at a decreasing rate with increasing excitation level. This phenomenon can be related to the damping properties of materials and structures which tend to increase with stress or acceleration level.

The amplification factor of a specimen, where the damping mechanism is defined as material damping, can be determined from Eq. (3-7);

$$Q = K\,(\text{response level})^{2-n} ,$$

where K is a constant for a given material, specimen geometry, and stress distribution. The constant n relates the specific damping energy D of a material to the stress level in the material (see Section 3.2, Effect of Sweep Methods). The value of n is dependent on the stress level in the part. For stress levels below 80 percent of the endurance limit of the material, n is 2.4; for stress levels greater than 80 percent of the endurance limit, n is 8. For viscoelastic damping n is 2 and Q is independent of excitation level.

It should be noted that the following equations are developed on the basis that the slope of the σ-N curve is constant from zero cycles to an infinite number of cycles. However, because of yielding and because of the existence of an endurance strength, the σ-N curve for most materials is horizontal in the low cycle region ($N < 10^4$) and in the high cycle region ($N > 5 \times 10^6$). The simplification of a constant slope permits a description of the exaggeration factor by a single equation but makes small inputs appear more damaging than they are. The use of exaggeration factors for items exposed to environments which create stresses below the endurance limit will therefore result in conservative tests. In addition, the reader should also be cautioned that there is a practical limitation to the amount of exaggeration, i.e., the increase in test level, which can be used. For example, it makes little sense to attempt to compress a test in time such that the increase in level will exceed the yield or ultimate strength of the material. A safe approach would be to limit the exaggeration of test level to not exceed the ratio of ultimate strength to endurance strength of the material in question (a value of 2 for many structural materials).

Exaggeration Factor for Sinusoidal Tests. In sinusoidal tests the stress level at a resonant frequency of a specimen can be defined by

$$\sigma = K_s QS,$$

where K_s is a proportionality constant and S is the excitation level. Combining with Eq. (3-30) results in

$$N(K_s QS)^b = c,$$

and from Eq. (3-7),

$$Q = K(QS)^{2-n}.$$

Thus,

$$NS^{b/(n-1)} = \text{constant},$$

which is the equation for the curve of excitation level vs cycles to failure. Substitution of T, time to failure, which is proportional to the number of cycles

divided by the natural frequency, results in the following relationship between excitation level and time to failure for two equivalent sinusoidal tests;

$$\left(\frac{S_1}{S_2}\right)^{b(n-1)} = \frac{T_2}{T_1}, \tag{3-41}$$

where the subscripts 1 and 2 denote test condition number. The value b ranges between 3 and 25 with a representative value of 9 for many structural materials. The value of n, as just discussed, ranges from 2 to 8 depending on material and stress level. The exaggeration factor can, therefore, have a widely different value depending on material and stress region.

A popular exaggeration factor [56] for use in testing complex electronic equipment is based on a stress-damping exponent of $n = 2.4$ and an endurance curve constant of $b = 9$, which results in

$$\left(\frac{S_1}{S_2}\right)^{6.3} = \frac{T_2}{T_1}.$$

This function, which is shown in Fig. 3-18, appears to result in either conservative or nonconservative test conditions depending on the stress ranges for the equipment in its service environments. The use of the value $n = 2.4$ is for stresses below the endurance limit and the use of the value $b = 9$ is for stresses between the endurance limit and the yield strength. They are thus contradicting

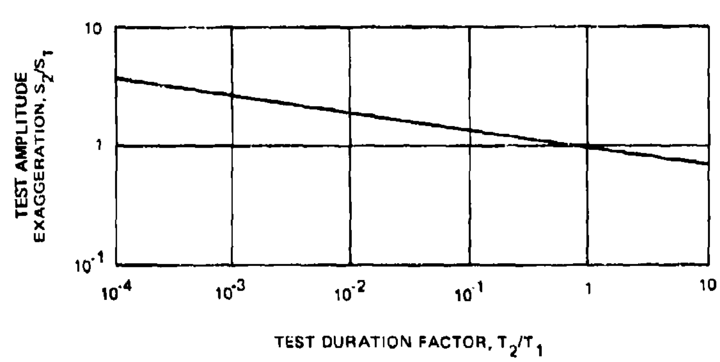

Fig. 3-18. Test exaggeration curve based on Eq. (3-41) for sinusoidal vibration with $\eta = 2.4$ and $b = 9$ (from Ref. 56). From *Shock and Vibration Handbook*, vol. 2, Fig. 24.26, p. 24-24; copyright 1961 by McGraw-Hill Book Company, Inc. Used by permission.

and the use of this factor will be conservative for S_1 below the endurance limit and nonconservative for S_1 between the endurance and yield strengths.

Exaggeration Factor for Random Test. The stress level at a resonant frequency of a specimen is defined as

$$\sigma = K_s(f_n Q W)^{1/2} ,$$

where W is the excitation spectral density. Combining with Eqs. (3-7) and (3-30) results in

$$Q = K(f_n Q W)^{(2-n)/2} ,$$

and

$$N W^{b/n} = \text{constant},$$

which is the equation for the curve of excitation level vs cycles to failure. Therefore, the exaggeration factor for random testing is

$$\left(\frac{W_1}{W_2}\right)^{b/n} = \frac{T_2}{T_1} . \qquad (3\text{-}42)$$

Note that the exponent here is different than for sinusoidal testing and therefore, for the same specimen, different exaggeration factors are required for random and sinusoidal testing except when $n = 2$, i.e., for viscoelastic damping.

Resonant Response

The simplest equivalences between sinusoidal and random vibration are based on equating the resonant response amplitude of an SDF system. For sinusoidal excitation the relative acceleration response at the natural frequency of the SDF system shown in Fig. 3-1 is

$$\ddot{Y} = Q \ddot{S}_0 ,$$

where $\ddot{Y}$ and $\ddot{S}_0$ are the peak response and excitation levels, respectively, and Q is the peak amplification factor. From Eq. (3-27) the mean square acceleration response for broadband random vibration is

$$\overline{\ddot{y}^2} = \frac{\pi Q f_n W}{2} ,$$

where W is the broadband acceleration spectral density. The waveform of the response is narrowband random with a Gaussian distribution of instantaneous amplitudes and a Rayleigh distribution of peak amplitudes.

Noting that the mean square value of a sinusoid is one-half of the squared peak value, the above relationships are combined to form an equivalence between a broadband random level and a peak sinusoidal level that will produce equal mean square responses:

$$W = \frac{Q \ddot{S}_0^2}{\pi f_n} . \tag{3-43}$$

Another common equivalence is formed by equating peak response amplitudes. Theoretically, from the equation for the Rayleigh distribution, the peaks of the random response can have infinite values. It is common practice in laboratory tests, however, to limit the peaks of the Gaussian distributed excitation signal to three times the root mean square value (this includes 98.9 percent of all the peaks in a Rayleigh distribution). By using this limiting value the peak responses from sinusoidal and broadband random can be equated to provide the equivalence relationship

$$W = \frac{2 Q \ddot{S}_0^2}{9 \pi f_n} . \tag{3-44}$$

Energy Dissipation

Equivalence relationships, similar to the fatigue relationships developed in a previous section, can be determined by equating the work done or energy dissipated by an object undergoing vibration.

In the following discussion the equation of motion of the system shown in Fig. 3-1 is used in the development of the equations for the energy dissipation of an SDF system with viscous damping constant c. Since energy dissipated is equal to the product of force $c\dot{Y}$ times distance $\dot{Y}dt$, the energy dissipated in one cycle is

$$\frac{E}{\text{cycle}} = \int_0^{2\pi/\omega} c\,\dot{Y}^2\,dt .$$

For steady state harmonic motion,

$$\dot{Y} = K \sin(\omega t - \alpha), \text{ for } K = \text{constant},$$

and therefore,

$$\frac{E}{\text{cycle}} = \int_0^{2\pi/\omega} c\,K^2 \sin^2(\omega t - \alpha)\,dt,$$

$$\frac{E}{\text{cycle}} = \frac{cK^2\,\pi}{\omega}\,. \tag{3-45}$$

Energy Dissipated During a Sinusoidal Dwell. For a single frequency dwell at the resonant frequency of the SDF system, the constant K in Eq. (3-45) is

$$K = Q\,\dot{S}_0 = \frac{Q\,\ddot{S}_0}{\omega_n}\,.$$

Substituting for K in Eq. (3-45) yields the energy per cycle as

$$\frac{E}{\text{cycle}} = \frac{c\,Q^2\,\ddot{S}_0^2\,\pi}{\omega_n^3}.$$

The energy dissipated E_c in a test time T_c is

$$E_c = \frac{c\,Q^2\,\ddot{S}_0^2\,T_c}{2\,\omega_n^2}. \tag{3-46}$$

Energy Dissipated During a Sinusoidal Sweep. During a sinusoidal sweep the work per cycle will vary as the response amplitude changes with excitation frequency. If the sweep is slow enough the velocity response vs frequency can be approximated by the steady state response function, i.e.,

$$\dot{Y} = K\,|H(\omega)|\,\ddot{S}_0\,,$$

where (from Eq. (3-28)),

$$H(\omega) = \frac{i\,\omega}{\omega^2 - \omega_n^2 - 2\,i\,\zeta\,\omega_n\,\omega}.$$

The energy dissipated in the sweep can be determined by summing the increments of energy in infinitesimal frequency bands

$$E_s = \int \frac{c\,K^2\,\pi\,dn}{\omega} = \int \frac{c\,\ddot{S}_0^2\,\pi\,|H(\omega)|^2\,dn}{\omega}$$

and

$$dn = \frac{\omega \, d\omega}{4\pi^2 \, |\dot{f}|},$$

where $|\dot{f}|$ is the instantaneous sweep rate. Thus,

$$E_s = \frac{c \ddot{S}_0^2}{4\pi} \int \frac{H(\omega)^2 \, d\omega}{|\dot{f}|}.$$

In the region of the resonance $|\dot{f}|$ can be considered constant and integration is performed as in Eq. (3-28);

$$E_s = \frac{c \ddot{S}_0^2 Q}{8 h \omega_n}$$

or

$$E_s = \frac{\pi c \ddot{S}_0^2 Q^2 T_s}{4 \omega_n^2}, \qquad (3\text{-}47)$$

where T_s is the time to sweep the half-power bandwidth.

Equating Eqs. (3-46) and (3-47) results in an equivalence between the time for sinusoidal dwell and the time to sweep the half-power bandwidths:

$$T_s = \frac{2 T_c}{\pi}. \qquad (3\text{-}48)$$

Energy Dissipation During Random Vibration. The power of the damping is equal to the time rate of the energy dissipation. Thus,

$$P = \frac{1}{T} \int_0^T c \dot{Y}^2 \, dt = c \overline{\dot{Y}^2},$$

where $\overline{\dot{Y}^2}$ is the mean square value of the relative velocity. The energy dissipated by the SDF system is

$$E_r = c \overline{\dot{Y}^2} T_r,$$

where T_r is the test time. Combining with Eq. (3-28) results in

$$E_r = \frac{\pi W Q T_r}{4 \omega_n} . \tag{3-49}$$

Equating Eqs. (3-46) and (3-49) results in

$$\ddot{S}_0 = \sqrt{\frac{\pi f_n W}{Q}} , \tag{3-50}$$

an equivalence relationship between sinusoidal dwell test level and acceleration spectral density test level.

Functional Performance

The definition of functional failure is that the performance of the equipment, whether it be electrical or mechanical, is degraded under the influence of vibration. The definition of failure is usually based on the degree of degradation. Included in the definition is the stipulation that there is no damage; that is, after the vibration ceases, the equipment performance returns to normal and there are no structural failures or permanent deflections. Simulation theory pertaining to malfunction is complicated by the numerous types of failures and phenomena which cause them.

Among the more common types of functional failures in electronic equipment are relay chatter, gyroscopic drift, microphonics in tubes and crystals, short circuiting, blurring of optics, etc. The phenomena associated with those various failures are as numerous as the failures themselves and include absolute acceleration, relative motion effects, absolute deflection, etc. An investigation [53] of the comparison of functional failures of typical aircraft electronic equipment subjected to random and sinusoidal vibration attempted to determine a correlation on the basis of experimental evidence. The conclusions of that study were that (1) for some equipment, no correlation will exist, (2) where complex systems are involved, the correlation, if it exists, will be so complicated that it will have to be determined by testing under both types of excitation, at which point the correlation is no longer needed, and (3) for systems where the functional failure is relatively simple in nature, e.g., relay chatter, a correlation may be determined by analytical means.

Summary of Vibration Equivalences

The equivalence relationships developed in the preceding sections are summarized in Tables 3-1 and 3-2. Table 3-1 lists the required test conditions (i.e., level and duration) for equivalence between the three standard test methods: sinusoidal sweep, sinusoidal dwell, and broadband random. These equivalences

Table 3-1. Equivalence Relationships Between Sinusoidal Sweep, Sinusoidal Dwell, and Broadband Random

Basis of Equivalence	Test Condition	Equivalent Test Methods		
		Sinusoidal Dwell	Sinusoidal Sweep	Broadband Random
Fatigue	Level	$\ddot{S}_0$	$\ddot{S}_0$	$W = \frac{KQ}{f_n} \ddot{S}_0^2$ $0.19 > K > 0.056$
	Duration	T_c	$T_s = K T_c$ $0.24 < K < 2.85$	$T_r = T_c$
Peak Response	Level	$\ddot{S}_0$	$\ddot{S}_0$	$W = \frac{2 Q \ddot{S}_0^2}{9 \pi f_n}$
Root Mean Square Response	Level	$\frac{\ddot{S}_0}{\sqrt{2}}$	$\frac{\ddot{S}_0}{\sqrt{2}}$	$W = \frac{Q \ddot{S}_0^2}{\pi f_n}$
Energy Dissipation	Level	$\ddot{S}_0$	$\ddot{S}_0$	$W = \frac{Q \ddot{S}_0^2}{\pi f_n}$
	Duration	T_c	$T_s = \frac{2 T_c}{\pi}$	$T_r = T_c$

are based on equating fatigue damage, resonant response, and energy dissipation in linear mechanical systems. Exaggeration factors, based on fatigue damage, are listed in Table 3-2. Exaggeration factors provide a relationship between one set of test conditions, level and duration, and another set of test conditions for the same test method. That is, the factors relate a test of low level and long time to a test of high level and short time. The terms used in these tables are defined below.

$\ddot{S}_0$	=	peak sinusoidal excitation level
W	=	acceleration spectral density
T_c	=	duration of sinusoidal dwell test
T_s	=	time to sweep half-power bandwidth of a resonance
T_r	=	duration of random test
K	=	constant
Q	=	peak amplification factor
f_n	=	resonant frequency
b	=	measure of slope of σ-N curve (see Eq. (3-30))
n	=	damping-stress exponent (see Eq. (3-7))
T_1, S_1	=	duration and level of sinusoidal test 1
T_2, S_2	=	duration and level of sinusoidal test 2
T_1, W_1	=	duration and level of random test 1
T_2, W_2	=	duration and level of random test 2.

Table 3-2. Exaggeration Factors for Sinusoidal and Random Tests

Test Method	Test 1	Test 2
Sinusoidal Dwell or Sweep	T_1, S_1	$S_2 = S_1\left(\frac{T_1}{T_2}\right)^{(n-1)/b}$
Random	T_1, W_1	$W_2 = W_1\left(\frac{T_1}{T_2}\right)^{n/b}$

Note 1. For viscoelastic damping $n = 2$. For low to intermediate stresses in elastic-plastic materials $n = 2.4$, and for high stresses $n = 8$.

Note 2. The value of b is in the range 3 to 25. However, a value of 9 is representative of many structural materials.

The relationships shown in Tables 3-1 and 3-2 should indicate that any universal equivalence between sinusoidal dwell and sinusoidal sweep and between

sinusoidal and random is nonexistent. The relationships are significantly different for different types of failure mechanisms. In addition, equivalences based on producing identical failures, such as fatigue, have widely differing values according to such factors as damping, stress level, and material properties.

3.5 Nonstandard Test Methods

The emphasis in the previous sections of this chapter have been on the characteristics of the three standard test methods and the relationships between them. The methods have been defined according to waveform and therefore the discussions have concentrated on the effects of the waveform on equipment. This section discusses two additional test methods which have waveforms different from the standard tests.

Combined Broadband and Narrowband Tests

As discussed in Section 3.3 the vibrations common in nature have a random amplitude with time. The standard test method for simulating these random vibrations is the broadband test where the excitation curve is defined by smoothly enveloping the peak values of the predicted or measured acceleration spectral density. A typical example of a test spectrum derived in this manner is illustrated in Fig. 3-19. In actual service the excitation spectrum will be broadband with several narrowband spikes superimposed. It is generally assumed that the center frequencies of the spikes can be such that a spike will occur at any frequency. This is the justification for enveloping the complex spectra with a smooth curve. The disadvantages to the wideband test are (1) the test specimen is subjected to a spectrum much more severe than any actual environment, and (2) the test requires much larger vibration shakers and power amplifiers than might otherwise be necessary. Two methods which offer improvements over the wideband test are the sine plus random and the sweep narrowband random.

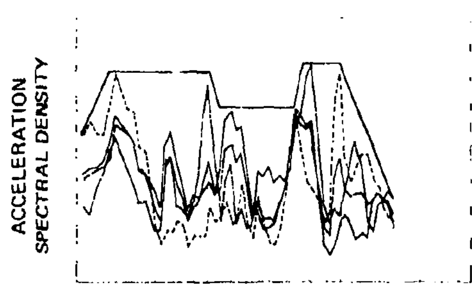

Fig. 3-19. Composite of typical measured flight vibration with enveloping curve.

The control methods and procedures necessary for the mixed sine and random test are similar to those required for the sweep narrowband random test. Therefore, unless the environment is characterized by sinusoids mixed with random there is no distinct advantage to the sine-random test. The sweep-random test retains the statistical character of the waveforms of most vibrations found in nature. Another advantageous property of the narrowband random test is that a given test level can be obtained using smaller shakers and power amplifiers than those required for a broadband test at the same level.

There are two approaches to narrowband testing. In one [59, 60], the test is based on equating the damage incurred by the wideband test to that of the narrowband test. This method uses one or more narrowband spikes and logarithmically sweeps through the frequency range. The rms magnitude of the narrowband spike is proportional to the square root of the center frequency of the spike. The equivalence relationship between this test and a broadband test is based on equating cumulative fatigue damage.

The other method [61] does not attempt to equate the narrowband test to the wideband test. Instead the justification for the test is based on the fact that the narrowband excitation is a better simulation of the actual environment. The method was developed from the results of a statistical evaluation of aircraft vibration which revealed that a single vibration spectrum could be described by three narrowband spikes superimposed on a relatively constant broadband level as shown in Fig. 3-20. The peak levels of the spikes represent the extreme expected level and are the values which would be enveloped in a more conventional approach. This spectrum is therefore a method of describing the environment in a more realistic manner than the conventional broadband description. Because the center frequencies of the narrowband spikes cannot be predicted it is necessary to assume that they could be any value. The test, therefore, which approximates this description of the environment requires that the center frequencies of the spikes be varied across the respective frequency ranges of each spike. This will verify the design for any values of the center frequencies which may be encountered in the service environment.

Simulation of Gunfiring Vibration

Of special interest, recently, is the vibration in aircraft resulting from the firing of high speed guns which fire at a rate of 20 to 100 rounds per second. The reaction forces from these guns are minimized by special isolation systems and generally transmit very little force to the aircraft structure. However, the blast pressure from the projectile charge cannot be eliminated. It is distributed around the exterior fuselage of the aircraft to varying degrees with the most extreme pressure levels in the vicinity of the gun muzzle.

A single pressure time history can be idealized as shown in Fig. 3-21 [62]. Expansion of this pressure time history in a Fourier sine series will reveal the frequency content, and will result in the coefficients shown in Fig. 3-22. The fundamental frequency of the harmonics is equal to the basic firing rate of the

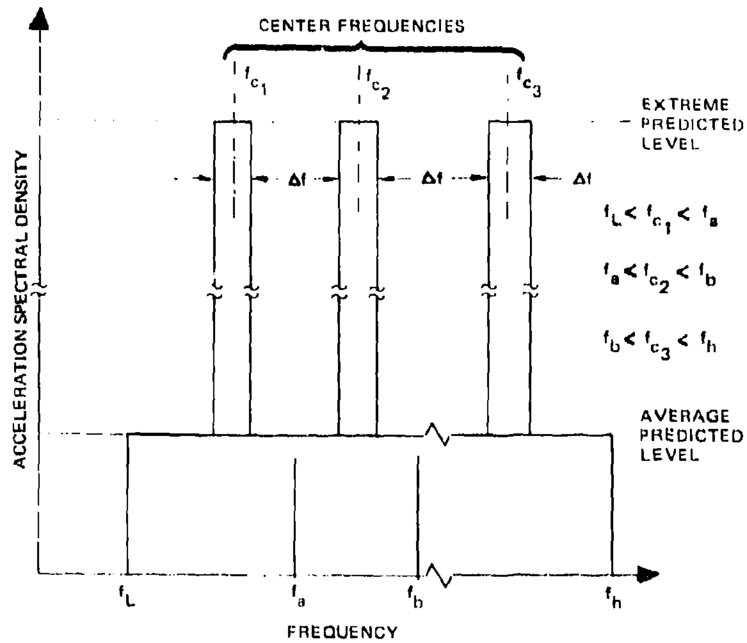

Fig. 3-20. Broadband narrowband description of aircraft flight vibration.

gun. Note that the amplitude of the harmonics remains high and fairly constant over a significant frequency range.

For firing rates of 100 Hz a forcing function would have significant levels covering a frequency range from 100 Hz to well over 2000 Hz. The structural vibration resulting from this forcing function will have the same character as the forcing function. The waveforms are described by line spectra at the harmonic frequencies with amplitudes and phase angles dependent on the dynamic transfer characteristics of the structure.

During any one burst, i.e., a series of rounds, the pressure and firing rate will vary because of differences in ammunition charge, variations in hydraulic pressure which regulates speed, mechanical tolerances, etc. In addition to the variation of firing rate within a burst of data, there is a greater variation of rate from one burst to another. A ±5-percent variation from the fundamental rate is typical.

Complex periodic waveform vibration also exists in tracked vehicles such as tanks and armored personnel carriers or in equipment with rotating or impacting machinery. It is likely that the fundamental periodic rate is also variable for

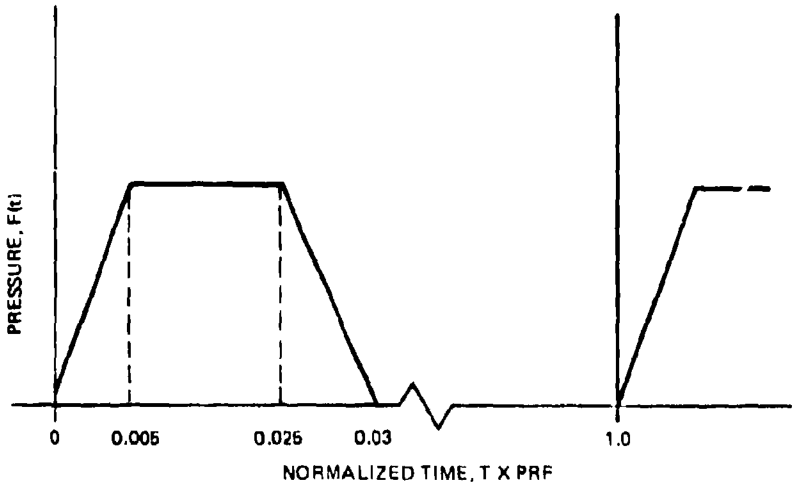

Fig. 3-21. Idealized blast pressure time history, normalized to period of firing rate frequency f_r.

these situations and that the techniques for simulation of gunfire vibration as discussed here are generally applicable.

Use of sinusoidal excitation to simulate gunfire vibration has some appeal since a periodic, or at least, almost periodic waveform is to be simulated and thus each harmonic of the waveform can be simulated in turn. The weakness of this approach is that the relationship between the effects of applying each harmonic individually and all harmonics simultaneously is difficult, if not impossible, to assess, particularly with respect to functional performance of the equipment. The rather obvious possibility of using the sum of the outputs of a number of oscillators, one for each harmonic, can be quickly discarded when the problems of amplitude and frequency control are considered. Use of broadband random excitation immediately permits simultaneous excitation of all harmonics of the environment. However, it is clear that a broadband level which in some undefined way is equivalent to the level of the harmonics must be very conservative in the frequency bands between the harmonics.

A unique method has been developed primarily for use as a simulation of the gunfiring environment [17]. This method utilizes a pulse train excitation source to be used in place of the oscillator or random noise generator in the test console. Several simulation requirements are immediately fulfilled. First, all harmonics are generated at once. Second, the deterministic nature of the waveform is achieved. Third, the phase relationships, even if incorrect, are at least not

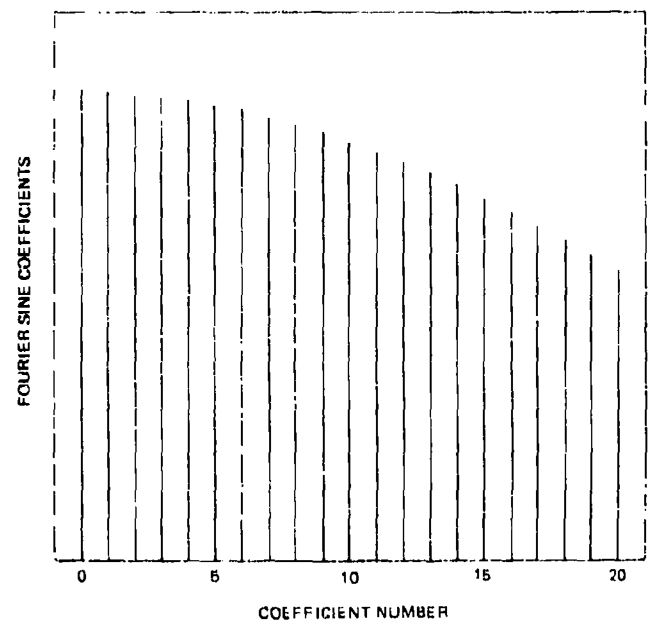

Fig. 3-22. Frequency content of blast pressure wave.

artifically controlled and would be repeatable for a given test setup. Fourth, variation of the pulse repetition rate would tune all harmonics correctly. Fifth, the test duration is immediately determined since real-time testing is achieved. This technique is discussed in more detail in later chapters dealing with implementation of tests.

CHAPTER 4
VIBRATION EQUIPMENT REQUIREMENTS

Previous chapters have described the various factors to be considered in the selection of a test method. The remaining chapters are concerned with the implementation and performance of the various methods. Of course, the factors discussed in these chapters should also be considered in the selection of the test and preparation of a test plan (see Appendix C). However, rigid contractual requirements specifying the test method are often set prior to such consideration. Even so, many of the factors discussed in these remaining chapters can be optimized within a given test method in such a way as to significantly influence the cost, time, complexity, and accuracy with which the test can be performed. This chapter presents equipment considerations which are applicable generally, regardless of test method, whereas Chapter 5 covers those factors related specifically to the various test methods and sets of conditions.

The basic elements of vibration test equipment are shown schematically in Fig. 4-1, linked together in the order in which mechanical or electrical signals flow through the system. The dotted line is intended to indicate the alternatives of manual or servo control while the parallel paths indicate that a load connection/support system or a vibration fixture, in the conventional usage of the term, may or may not be necessary, depending on the configuration of the test item.

The following sections contain discussions of the major factors with respect to all the elements shown in Fig. 4-1 except the waveform generation equipment and most of the control system equipment, which are discussed in Chapter 5.

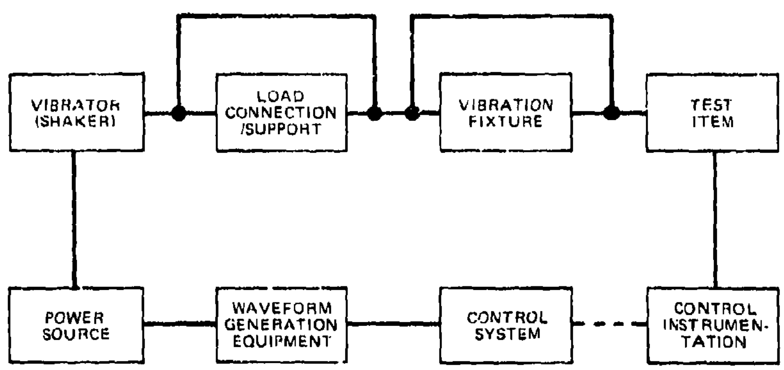

Fig. 4-1. Basic elements of vibration test equipment.

Information which is normally available from manufacturers of vibration test equipment is not included.

4.1 Vibration Excitation Systems

A vibration excitation system is defined here as consisting of the driving or power source, the vibrator (shaker) and, where applicable, the external load connection/support system. Driving source/vibrator systems may be classified into three categories characterized by their nominal maximum test frequency: (1) low frequency (50 to 60 Hz), (2) intermediate frequency (500 to 900 Hz), and (3) wideband (2000 to 5000 Hz). The material in Chapters 4 and 5 is applicable generally to the use of either of the last two categories, although the discussion is directed mainly to the use of wideband, electrodynamic systems. If one excludes product assurance testing, most current testing is performed using wideband systems.

Vibration System Types and Characteristics

The low-frequency vibration system is exemplified by the inertially driven reaction-type exciter. It is available in a wide range of load-bearing capacities, has test frequency capabilities in the range of about 5 to 60 Hz, and is limited to low displacements and sinusoidal waveforms.

Intermediate-frequency systems are typified by the hydraulic shaker. The driving force is derived directly from a hydraulic power supply, with very large force ratings possible. The application of the force may be electronically programmed, thus permitting sinusoidal, complex, or random waveform testing within the system's frequency capability. Test frequency capabilities may range from as low as 0.1 Hz to an upper limit of 500 to 900 Hz. Relatively large displacements are possible at the lower frequencies, with several inches commonly available and a few man-rated systems providing several feet.

Wideband systems are comprised of electrodynamic shakers driven by electronic power amplifiers. Older systems are limited to the frequency range of 10 to 2000 Hz, but systems have been available for several years with upper limits of 3000 to 5000 Hz. Maximum peak-to-peak displacement capabilities range from 0.5 to 1/in.

Vibration System Capacity

For any given test, the ability of the vibration system to produce the desired waveform over the desired frequency range at the required level is of prime importance. It is obvious that use of a system with insufficient capacity is likely, at best, to result in only a partial achievement of test objectives. However, even if the selection of a facility is based on nominal force requirements which appear to be adequate, there are adverse results which may be expected. For example:

1. The ubiquitous problem of designing an adequate fixture is likely to be compounded by the weight constraint imposed by the force limit.

2. There is likely to be impairment of the ability to program and control test levels due to the combined influences of the electromechanical characteristics of the shaker system and an unfavorable mass ratio of test object, fixture, etc., to shaker armature.

As a general rule, the greater the capacity of the vibration system, the less pronounced are these effects. Conversely, if equipment costs, availability, or other considerations make necessary the use of a marginally powered system, test performance difficulties should be anticipated and compromises that will probably be required in fixturing, test quality, and test time should be recognized and accepted. It is unfortunately true that the adequacy of the excitation system can rarely be determined rigorously prior to testing the actual test item at full test level.

In addition to the system's force rating, the maximum stroke of the shaker armature must be considered. First, depending on the weight of the test mass and the stiffness of the armature flexures, the stroke available during vertical testing will be less than the rated stroke because of static armature deflection due to the test load. This problem is avoided on some recently developed shakers by incorporation of a device which recenters the armature as load is added. Some older shakers can be modified to provide this repositioning capability [63]. Second, since the maximum stroke of some shakers is limited to a 0.5-in. double amplitude, acceleration levels specified at lower frequencies may not be attainable because the resulting displacement exceeds the stroke limit.

Load Connection and Support Systems

Load connection and support systems include those devices employed to couple the shaker armature to and to support the dead load of the test item and, where applicable, the test fixture in which it is mounted. See Fig. 4-1.

The need for such devices arises generally when one or more of the following situations obtains:

1. The test item is large or massive.

2. The orientation of the test item with respect to gravity forces must be maintained, regardless of axis of vibration excitation.

3. The test item is to be tested along each of three orthogonal axes.

4. The deadload or moment cannot be supported by the shaker.

5. The dynamic moment (overturning moment) cannot be reacted by the shaker.

6. Motion normal to the excitation axis, i.e., crosstalk, must be minimized.

7. The test item is to be vibrated at one or more locations which are not "normal attachment points" and no "fixture" is employed.

8. A single, less-expensive fixture can be employed for all test axes in conjunction with such a system.

The design of such devices must fulfill, to the greatest possible extent, the following requirements:

1. Minimize distortion of the desired vibration waveform at the test item.

2. Minimize effects on the response of the test item to the required vibration excitation.

3. Prevent, or at least, not amplify crosstalk motion.

4. Minimize the reduction in vibration excitation system capacity.

5. Be convenient to use.

6. Protect the shaker armature from excessive loading.

These devices can be grouped into three basic categories which are discussed in the following subsections. These categories are (1) linkages between the shaker and the test item, the test fixture, or the slip plate through which the excitation is transmitted; (2) slip plates which provide a mounting platform in a horizontal plane to be driven in a horizontal direction; and (3) devices used to support the dead load of the test item. Requirements for design of categories 1 and 2 have much in common with the design of vibration fixtures discussed in Section 4.2. In particular, the requirement for preloading of bolted connections [64] and for care of mating surfaces must be observed for satisfactory performance.

Linkages. Linkages, in the present context, serve as transition structures to transmit the shaker motion or force toward the test item. Most shaker armatures are circular with one or more concentric bolt circles for attachment. The linkage must be a rigid, efficient structure to provide the transition from the circular armature to (1) a line for attachment to a slip plate, (2) a larger area for attachment to a fixture, or (3) a smaller area for attachment directly to a test item. Types 1 and 2 are completely rigid, whereas type 3 may be flexible in shear or bending. This flexibility is required to protect the shaker from excess loads when the test item center of gravity is offset from the line of force or when significant shear and flexural response loads can be predicted. Figs. 4-2, 4-3, and 4-4 show typical linkages of the three types mentioned. Tolleth [65] discusses the design of linkages to drive slip plates. Such linkages are usually attached to the shaker and the edge of the slip plate through bolts in tension. However, for the slip plate, it is structurally more efficient to apply shear loading by, for example, using cylindrical expansion sleeves in aligned holes through linkage and plate normal to the thrust axis. The latter approach has the added advantage of speeding up the process of changing shaker orientation since only loosening or tightening is required rather than complete unthreading or threading. Obtaining desirable frequency response characteristics for the linkage-slip plate combination is difficult, but careful design techniques yield reasonable performance. The authors know of a few instances where an integral linkage/slip plate has been fabricated from a machined casting; however, the degree of

Fig. 4-2. Typical armature-to-slip-plate linkage.

improvement over the conventional configuration is unknown and may not justify the significant increase in cost.

The linkage shown in Fig. 4-4 incorporates an "X" flexure which permits the missile to "pitch" with negligible moment applied to the armature. Such linkages developed from the rather unsatisfactory experience with the use of mechanical fuses, i.e., rods threaded at each end and "necked down" in the middle. These linkages tended to absorb too much shaker capacity because of a low longitudinal resonant frequency and fractured rather frequently.

For small loads mounted on the shaker table where repetitive testing of successive items is required, attempts have been made to speed up the installation-removal cycle by using vacuum [66] or liquid film mounting [67] techniques with some degree of success. These methods may be useful for low-mass items where acceleration forces, particularly at the lower frequencies, are not too large.

Slip Plates. The majority of vibration tests require the test item to be vibrated in each of three orthogonal axes, typically the vertical, longitudinal, and lateral axes of the test item. When the test item is mounted in a fixture, it is generally economical and sometimes mandatory, if the orientation of the test

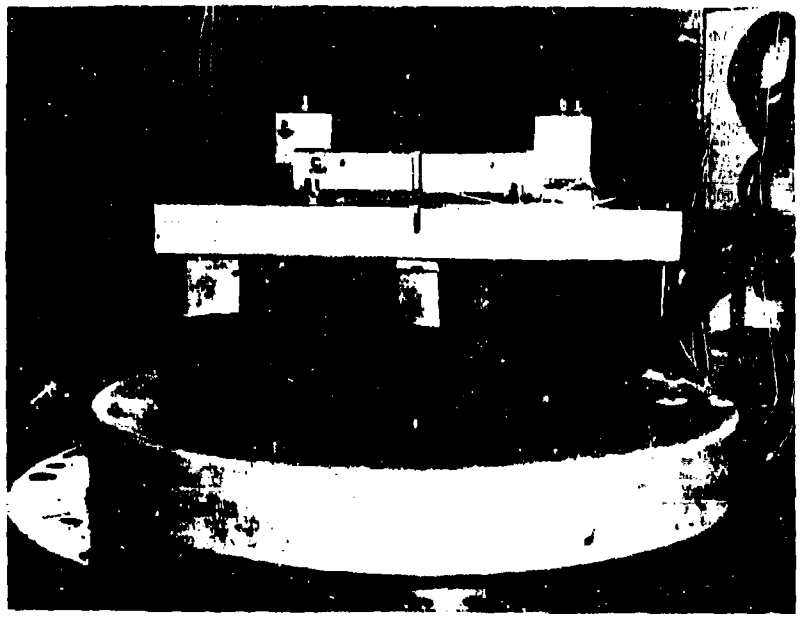

Fig. 4-3. Typical armature-to-fixture linkage.

item is significant (e.g., when vibration isolators are part of the test item), to use a slip plate for testing in the two horizontal axes.

The more common of two basic types of slip plates is shown in Fig. 4-2. The slip plate floats on a thin oil film on a granite block [68-70]. The shaker mounting hole pattern is usually repeated in the plate to permit use of a single fixture. The problem of large fixture overhang for at least two test axes is minimized, and a relatively large overturning moment, due to vertical offset of the center of gravity from the thrust axis, can be tolerated. However, flatness tolerances on the plate and top surface of the block are tight; protection of these finished surfaces during use, handling, and storage is required, and alignment is critical if the necessary film tension restraining forces are to be maintained. It is obvious also that the maximum acceleration level which can be applied to a given test mass is reduced due to the increased load.

All but the last of the above difficulties can be eliminated by the use of a test bed riding on shoes sliding in restraining slots and lubricated by high pressure oil feeds. Such equipment is commercially available and will withstand very high off-axis loads successfully. However, these systems are considerably more expensive than slip plate installations.

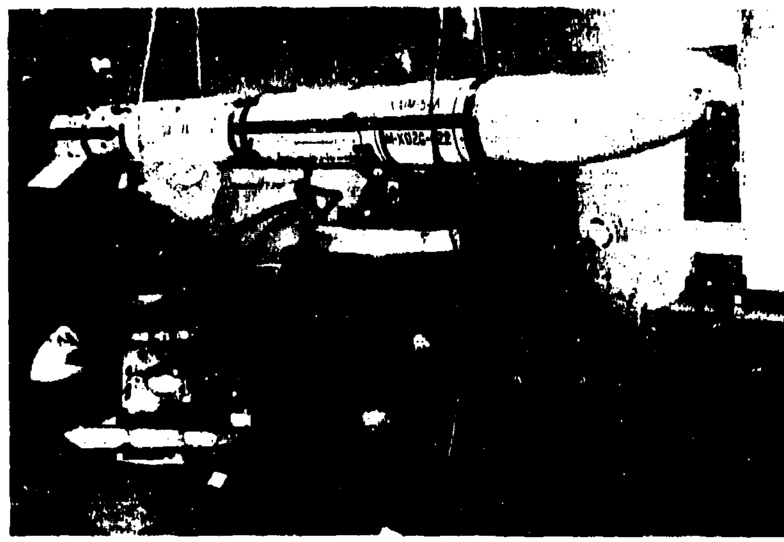

Fig. 4-4. Typical armature-to-test-item linkage.

Prior to the development of slip plates, support of test objects for testing in the horizontal axes was provided by either a set of rollers under the fixture or plate or an array of vertical flexures attached to a relatively inert base [71]. The problems encountered with resonances, crosstalk, and rattles in these types of support are readily imagined.

Static Load Support. Tests of large or massive items, except when slip plates are employed, generally require a device to react the static load of the item. If the item is mounted on top of the shaker, it may be necessary to reduce the stress on the armature flexures and/or preserve the full positive and negative displacement capability of the shaker. (See page 105 also.) When the shaker axis is horizontal, static shear and bending moment on the armature must be prevented. In either case, the static load support must be "soft," i.e., the natural frequency of the suspended mass (assumed rigid) in the excitation axis must be low compared to both the lowest resonant frequency of the suspended mass and the lowest excitation frequency. The support device must also be capable of position adjustment for alignment of the test object with the shaker.

A variety of support mechanisms such as air bags, elastic cables (e.g., shock cords), and metal springs in combination with hoists, jacks, etc., may be successfully used for static load support. For example, see Fig. 4-4.

It is generally advisable, in those tests where static load supports are necessary, to provide a backup support or "preventer." This support is supposed to catch and support the test item in case of failure of the main support system.

The ropes visible in Fig. 4-4 have this purpose but fortunately have yet to be proven adequate.

4.2 Vibration Fixtures

There are many factors to be considered in the design of vibration fixtures. In addition to the general considerations such as weight, configuration, and cost there can be requirements on stiffness, durability, and simplicity. In a typical program the fixture requirements appear in three forms: The project office desires fixtures that cost nothing, can be built overnight, will accommodate ten units simultaneously, can be used for holding fixtures when not in use for vibration, and can be used on other programs. The specification people want fixtures that have infinite stiffness and zero cross-axis response. The test laboratory wants a fixture which weighs nothing, will allow all three test axes to be performed on the slip plate, and can be switched to another axis by removing and replacing one bolt. A poorly designed fixture is usually the result of excessive influence by any one of the three groups. A good or adequate fixture will possess a balance of all of the above factors. It is the job of the fixture designer to place these sometimes contradictory wishes in perspective and derive his own set of design requirements that will allow the design of an optimum fixture. The following sections outline the primary considerations in fixture design. References 72 and 73 contain more detailed discussions on this subject.

Weight

A controlling parameter in the fixture design is the maximum allowable weight. This weight is a function of the available excitation force, vibration test requirements (i.e., level, frequency range, and method of control), and moving mass, i.e., weight of test article, shaker armature, slip plate, etc. As a first approximation the allowable weight should be calculated on the basis that the moving mass remains rigid throughout the frequency range of the test. For example, a 10,000-lb shaker would be capable of exciting a moving mass of 1000 lb at a sinusoidal level of 10 g's. However, because the moving mass will respond dynamically to the excitation, there will be frequencies where additional force will be required to maintain a given acceleration level. The amount of additional force depends on the dynamic characteristics of the complete test configuration and the method of test control. A method which utilizes a single accelerometer location for level control will generally require more force than an averaging or signal selection method. A test engineer can usually estimate the force requirements on the basis of past experience with similar tests. However, a good rule of thumb is to multiply the test article weight by ten when using single-point control and by two when using averaging or signal selecting techniques. This weight is then used as follows to calculate the allowable fixture weight:

$$\text{Fixture weight} = \frac{\text{Force rating}}{\text{Test level}} - \text{Test weight}\ ,$$

where

Test weight	=	Total weight of moving mass (i.e., armature, slip plate, test article, etc.) minus the fixture weight. (Test article weight multiplied by 2 or 10 as defined above.)
Force rating	=	Force rating of shaker (peak or rms, respectively, for sine or random)
Test level	=	Acceleration level of test control (in units corresponding to type of test, i.e., peak for sine, rms for random).

Stiffness

The stiffness requirement of a vibration fixture is different for different types of tests. For engineering evaluation tests where the dynamic characteristics of the test article are to be determined, the stiffness of the fixture is extremely important. It must be controlled so that information derived in the test can be properly interpreted. Stiffness is also important for tests where the fixture is to simulate the mounting structure of the test article [30]. For testing of light components and off-the-shelf equipment to military standards requirements, consistency is important, and therefore so is the stiffness of the fixture. For the large majority of tests, however, the requirement on the fixture stiffness should be the least important factor of all the design requirements. Unfortunately, it is often treated as the most important. Efforts are directed toward making fixtures as stiff as possible with the ultimate objective of attaining equal in-phase motion at all test article attachments throughout the frequency range of the test. This effort usually results in expensive, heavy, and limited-use fixtures. Most important, environmental simulation is degraded. This is evidenced by the fact that vibration fixtures, especially those designed for stiffness, have no resemblance to the actual foundation structure of the equipment. The ultimate in good simulation would be to provide a fixture that simulates the equipment mounting impedance and to control the test in a manner which would allow the article to influence the base motion. The technical difficulties and expense are at present too great for such a test. It can be approached, however, through the use of excitation and control techniques discussed in Chapter 2 and by not using fixtures which are too stiff when compared to the mounting structure of the test article. This last statement is negative in that it advises not to use a certain fixture design rather than suggesting a requirement. This is intentional because specification of any stiffness, even one that approximates actual conditions, is often too binding a constraint on fixture design.

Occasionally fixtures must be made unusually stiff to enable the achievement of full test level. This may come about when the test requires the use of a fixture that has a configuration that structurally provides a low-pass filter between the

excitation and the control accelerometers. As an example, the testing of a large object on the head of a shaker where the fixture has a significant amount of overhang often results in an inability to achieve test level above the frequency associated with the fundamental bending frequency of the fixture. The problem can usually be solved or at least reduced if the stiffness of the fixture is increased.

Material and Method of Construction

The factors which influence the selection of a material are weight, cost, availability, and ease of construction (e.g., machinability). Aluminum and magnesium are the most popular for fixtures. Aluminum is generally more desirable because of its strength, especially for fixtures which will have repeated use. Method of construction, that is, welded, bolted, or cast, depends on two factors: cost and schedule. Bolted fixtures may be the cheapest to manufacture but can cause problems in test control if improperly designed. Cast fixtures usually are the most expensive (an exception to this general rule is discussed in Ref. 74) and therefore difficult to justify except for special situations. The most economical and useful fixtures usually are both welded and bolted. Bolts should always be preloaded [64] to insure against separation of parts, a highly nonlinear phenomenon which can cause problems in test control. Use of laminated fixtures is discussed in Ref. 73.

Miscellaneous Considerations

If at all possible, fixtures should be designed to allow three-axis testing on either the head of the shaker or the slip plate. This is to eliminate the need to rotate the shaker from horizontal to vertical or vice versa In addition the design should minimize the effort required in switching axes; the unit should be easy to remove from the fixture and the fixture from the shaker or slip plate [66]. It is also desirable, although seldom possible, to be able to detach the fixture without removing the unit. For testing large or massive items which require large fixtures, integrated slip plate/fixtures should be considered [73].

If a fixture is to be used repeatedly it is desirable to use threaded steel inserts at all locations that require continual removal and replacement of bolts. In addition, bearing surfaces which are under high preload stresses may require special attention for repeated loadings to prevent galling (i.e., heat treatment, material or chemical coatings).

4.3 Instrumentation and Control

Transducer Characteristics, Location, and Mounting

If the test to be performed is to yield any meaningful results, one must be able to identify the point, or points, for which vibration levels are to be defined

and controlled, whether the specified "inputs" be in terms of displacement, velocity, acceleration, a combination of these, or force. Although the exclusive use of accelerometers has become fairly common in recent years for tests where a mixture of displacement, velocity, and acceleration levels is required, a few observations concerning the first two types of transducers are in order since, for an occasional test at lower frequencies, they represent the optimum instrumentation approach.

Displacement Transducers. Practical displacement measurement techniques can be classified conveniently into three basic categories according to the parameters used to convert motion to its desired analog: resistive, capacitive or inductive, and optical. Since all depend upon the change of a parameter proportional to the absolute displacement of the test mass, part or all of the transducer (or the observer, in one case) must be supported rigidly, or isolated, to provide an inertial reference against which motion of the test mass is measured. It is obvious, then, that their useful frequency range is dependent on this support structure as well as on the limitations inherent in the design of the transducer itself. The three types, along with additional factors to be considered in their application, are as follows:

1. Potentiometric or slide-wire. A regulated excitation voltage is applied across the potentiometer allowing pick-off of a signal proportional to displacement by use of a slider connected by a mechanical linkage to the test mass. Other factors which should be considered are the inertial loading of the slider, linkage, and its attachment; the degree of resolution (determined by slider dimension, fineness of potentiometer wire, and total resistance); and noise effects due to transient changes in slider contact pressure.

2. Capacitive or inductive. A regulated excitation voltage is applied and a change in capacitance [75] or inductance due to relative motion, respectively, of either an equivalent capacitor plate or a piece of ferromagnetic material attached to the test mass permits the generation of a signal proportional to displacement. Loading is minimal usually but linearity of output and low resolution, commonly resulting from the fact that the total parameter change is relatively small, are additional factors to consider.

3. Optical. There are three techniques which may be used; in one, the specular reflection at a relatively sharp boundary between a light and dark area on the test mass is tracked through optics by an electronic servo which generates an output signal proportional to displacement [76]. No loading at all occurs, and both resolution and frequency response nominally are far better than for other systems. However, its rigid mounting is much more difficult due to sensor size and mass and is further complicated by the placement limitation imposed by the focal length of the optical system. A recently developed method involves the use of the laser interferometer. For details see Refs. 77, 78, and 79. The third method involves the use of the "optical wedge" (Fig. 4-5) and can be applied only to the case of pure sinusoidal motion and for displacements of 0.05 in. (peak-to-peak) or more. Since its effectiveness depends partially upon the

phenomenon of retentivity in vision, the wedge cannot be used at frequencies below 14 to 16 Hz. The minimum resolution of 0.05 in. double amplitude (DA) represents rapidly increasing acceleration with increasing frequency, i.e., proportional to frequency squared. For example, at 125 Hz, the acceleration is 40 g peak. For additional information, see Ref. 80.

Velocity Transducers. The operation of the velocity transducer in general depends on the inertial displacement of a coil in a magnetic field. As the coil, which is connected into the sensing and measurement circuitry, cuts links of the magnetic field, an electromotive force is generated which is proportional to velocity. Because it usually requires a self-contained magnetic field, the velocity transducer is relatively heavy. Therefore, its use should be reserved for applications where fairly high loading of the test mass can be tolerated.

Accelerometers. Piezoelectric types of accelerometers have been most commonly used in recent years because they are relatively light, are available with a fairly wide range of sensitivities, and are most easily used with modern commercial electronic programming and control equipment. Their outputs are derived by imposing an inertial force on a piezoelectric crystal, with the strain thereby created generating an electrical charge proportional to acceleration. Their high-frequency response is quite good, with most being usable to well above 2.0 kHz and some having inherently accurate response up to 10 kHz. Nominal limits on their upper frequency range are determined by accelerometer resonance characteristics [81]. However, it should be noted that the mounting method is likely to be the major factor limiting the frequency range over which useful data may be obtained. Piezoelectric accelerometers generally have a limited low-frequency response, with increasing signal degradation below 10 to 15 Hz.

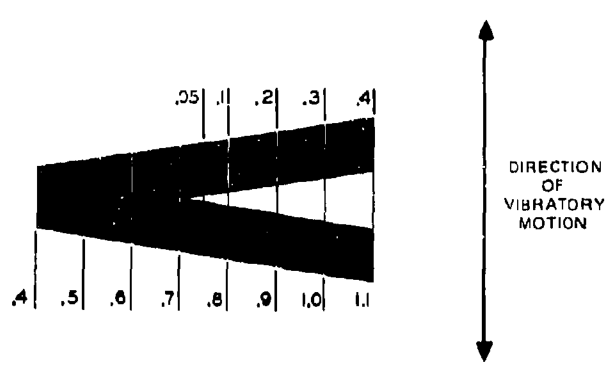

Fig. 4-5. Typical optical wedge.

For low frequency tests, or when greater signal sensitivity is required, the strain-gage accelerometer may be preferable. There are two basic types: wire resistance and piezoresistive. They both provide direct current (zero frequency) response; however, the upper frequency response is limited to the range of about 50 to 300 Hz for wire types and they are relatively heavy and susceptible to damage if their acceleration range is exceeded. The piezoresistive type is much lighter, can provide much higher frequency response, and is somewhat less susceptible to overacceleration damage [82].

Location and Mounting. As has been noted, definition of control points should be an essential part of test design. Adequate test planning, then, should lead to provision in the fixture design for convenient mounting of transducer(s) at the point(s) for which test levels are to be controlled [83]. For the test where control must be based upon an input modified, or limited, according to the response at one or more points on the test object itself, the selection of the location and the type of response transducers requires careful consideration. For example, if a point at which the response must be monitored is likely to flex during vibration, there are two basic problems in selecting an appropriate transducer regardless of mounting method: (1) the accelerometer must be light enough to minimize loading effects sufficiently, and (2) its sensitive element must be well-enough isolated from its case so that strain imposed on the latter by the test structure does not induce s. ·ious signals in the accelerometer output. It should be noted that some commercially available miniature accelerometers are particularly sensitive to case distortion.

Threaded studs (or screws) and cement are most commonly used for mounting accelerometers. If no precautions are taken, a threaded attachment will often introduce electrical noise into the output of the accelerometer due to grounding of its case. This problem can be avoided by using isolated mounting studs which are commercially available with good mechanical response characteristics. For threaded attachments it is important to use the installation torque recommended by the accelerometer manufacturer to avoid deviation from the calibrated sensitivity. The effects of mounting variables on the accelerometer's performance are described in Ref. 84.

The most popular material for cemented attachments is Eastman 910 because it is easy to use and attains full strength within a few minutes after application. If it is applied properly to a clean, flat surface, the resulting bond is adequate if instantaneous acceleration levels do not exceed 60 to 75 g's and if it is not exposed to temperatures outside the range of about 0 to 75°C. It is often necessary to interpose a thin fiberglass pad between accelerometer base and mounting surface (cementing both) to avoid ground-loop noise. Frequency response in either case is surprisingly good: even with the insulating pad it is satisfactory up to 3 kHz [85]. If the mounting surface is not smooth or flat, dental cement can be used with satisfactory results. At least 45 min must be allowed for curing at ambient temperature with more time required if test levels greater than about 10 g's are expected; curing time can be reduced by applying

heat carefully. Double-back, pressure-sensitive tape is sometimes used for accelerometer mounting; it is not recommended for test levels exceeding 4 to 5 g's or at frequencies greater than 500 Hz [85,86].

Other miscellaneous mounting techniques involve the use of a thin layer of wax or a permanent magnet. Good frequency response is claimed but no evidence of independent confirmation has been found in the literature.

In addition to assuring satisfactory attachment of accelerometers, it is necessary to secure their cables to prevent whipping which is likely to induce spurious signals in the cables, at the connectors, or as a result of strain imparted to the accelerometer case. It is recommended that all cables be taped or lashed as required to prevent their motion relative to the test mass and accelerometers.

Averaging

As was noted in Section 2.2, control of the test level to the average, either absolute or power, of two or more transducer signals has become common practice [31]. The synthesis of a control signal which has the properties of the desired average is often achieved by the use of a commutating device known as a time division multiplexer (TDM), whose function is illustrated in Fig. 4-6. The output of the TDM consists of sequential time samples of each signal. In normal operation during sinusoidal tests, commutation is synchronized with the excitation frequency so that each successive sample contains one period of motion. For use in random or complex-wave testing, the dwell or gating time is adjustable over some range.

Failure to observe certain precautions in the use of the TDM [87] can cause significant errors. Basically, these errors all stem from the fact that the spectral characteristics of the TDM output signal will almost always differ, to greater or lesser degree, from the spectral characteristics of the average of the individual signals. Significant errors stem mainly from either of two situations: (1) the polarity of one or more of the individual signals is inverted from the remainder; and (2) significant amplitude differences exist between adjacent inputs to the TDM.

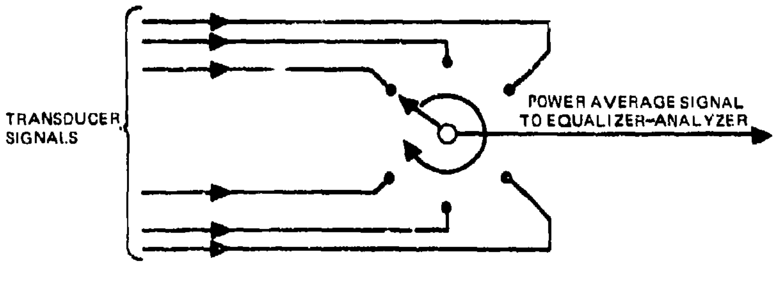

Fig. 4-6. Power averaging of random signals by commutation (time division multiplexing).

Relative inversion of signal polarities resulting from orientation of the transducers can be removed either by physical relocation or by electrical means. When relative inversion or large amplitude differences exist because of resonant response, modification of control or averaging methods may be required.

The potential errors in the use of the TDM fall into the following categories with differing implications regarding test quality:

1. The synthesized signal is not the average (absolute or power) of the individual signals, creating overtest or undertest.

2. The vibration control system reacts erroneously to a correctly synthesized average signal, creating overtest or undertest.

3. Independent analysis of the synthesized signal for test condition documentation is incorrect, indicating overtest or undertest even though test performance was correct.

4. Independent analysis of the synthesized signal is incorrect to the same degree as the control system and thus validates an erroneous test.

It will be seen that it is easy to commit each category of error.

Since the adverse effects differ for sinusoidal, complex, and random waveforms, they are discussed separately below.

Sinusoidal Waveforms. For the simple unfiltered sinusoidal test, no unusual precautions are needed. However, if fundamental control is attempted with relative input polarities reversed (or significant relative amplitude differences) and using insufficient TDM dwell times, very large errors in control level will occur.

For fundamental control a tracking filter is used to remove the unwanted distortion from the accelerometer signals. However, the output of the TDM contains distortion products (sidebands) due to step changes in the signal level when the TDM switches from one input to the next. The effect on the control signal is most pronounced when adjacent channels of the TDM have opposite phase or large variations in amplitude. These sidebands must be passed by the filter if a reasonably accurate absolute average of the fundamental components of the control signals is to be obtained. The obvious solution to the problem is to increase both the TDM dwell time T (by using the random mode) and the filter passband B so that the relative distortion resulting from TDM switching is reduced and most of the desired sidebands are contained within B. Unfortunately, as will be noted later, servo time constant and test sweep rate considerations impose severe constraints on the maximum permissible T.

Usher [87] shows that for the extreme case, where alternate TDM channels are 180° out of phase, the BT product must be 10.6 or more for about 80-percent accuracy. It should be noted that the inaccuracy will result in overtest because the output of the tracking filter will always be less than it should be.

When T is increased the servo time constant must also be increased to avoid "hunting" due to amplitude variations at switching and consequent modulation of the input to the power amplifier. The modulation will occur at low frequencies caused by relatively large T. To reduce this effect, the servo time constant

must be made large compared to T. However, an upper limit is imposed on the servo time constant because the servo response must be fast enough to correct unwanted test amplitude variations with sweep frequency. Adjustment of the servo time constant required to limit modulation to an acceptably low value depends on the specific equipment being used. Usher provides an example (using a 20-Hz filter) where a time constant of 27.3 sec is required to limit the modulation to 5 percent where the control signal is derived from multiplexing two channels (one at zero amplitude) with a T of 0.084 sec. This is an obvious worst case, but it does illustrate graphically the difficulty of test performance in this mode.

It should be noted that the foregoing problems can be eliminated by the use of multiple tracking filters. Each control transducer signal can then be filtered and the filter outputs connected to TDM input channels.

Complex Waveforms. For complex waveform tests, if multiple filtering and control of individual frequency components are attempted, the factors described above are applicable (with obvious complications in selection of dwell time, filter bandwidths, and servo detection times). If the broadband output of the TDM is used for test control with inverted input polarities, control quality is not likely to be degraded. However, subsequent analysis of the recorded control signal to define spectral test levels may indicate spectacular errors at some frequencies. The degree of indicated error will depend upon the TDM dwell time, analysis filter bandwidth, and the relative spacing of the input frequencies comprising the excitation waveform. If relative signal polarities were not correct (or are unknown) for the test, it is recommended that definition of control levels be based on the analysis of individual control transducer signals and calculation of their spectral averages.

Random Waveforms. As was noted in Section 2.2 under "Random Test Level Control," random waveforms must be power-averaged by deriving a signal whose spectral density is equal to the average of the spectral densities of the individual signals. Figures 4-6 and 4-7 illustrate schematically two means by which such a signal can be synthesized.

Decorrelation Method. The method shown in Fig. 4-7 results in delaying each signal with respect to the other signals so that they are decorrelated.

Since the signals originate from a common noise source, they are normally well correlated. The spectral density of the sum of independent or uncorrelated random signals is equal to the sum of the spectral densities of the individual signals. Thus, the desired power average signal is obtained by mixing the successively delayed signals and dividing by the square root of the number of signals. The schematic diagram of Fig. 4-7 may be readily implemented by recording and reproducing the individual signals (except the first) on a magnetic tape recorder to achieve the desired delay time. For a large number of signals, this may strain the available recording capacity since the additional tape recorder channels (record and reproduce) required for n transducer channels is

$$\text{Additional channels} = 1 + \sum_{i=1}^{n-2} i \tag{4-1}$$

This equation assumes that a recording of the power average signal is required as well as the normal requirement for recording the individual signals. The delay time between the recorded and reproduced signals occurs by reason of the time required for the tape to travel from the record to the reproduce heads. It is therefore a function of the physical design of the particular tape recorder and the tape speed selected. A commonly available recorder provides a 200-msec delay at a 15-ips tape speed.

Conflicting desires enter into selection of the appropriate time delay. The decorrelation of the signals is improved as the time delay increases. On the other hand, as the time delay and the number of channels increase, the potential time for the vibration control system to sense a change in level also increases. This problem is primarily of importance during the process of initial equalization and coming up to full test level and can be minimized by changing the master gain

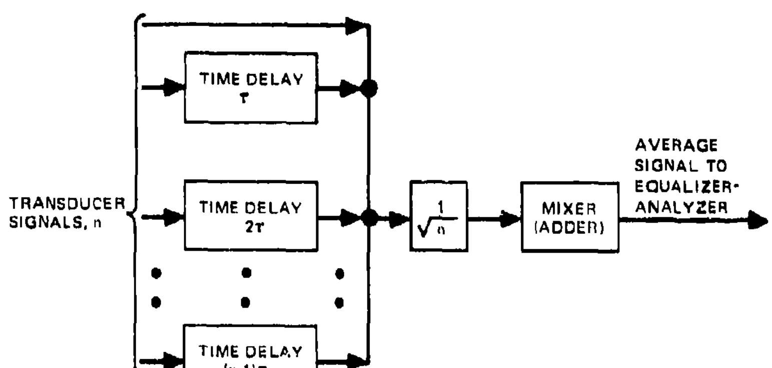

Fig. 4-7. Power averaging of random signals by time delay (decorrelation method).

control and the equalizer settings somewhat more slowly than usual to allow for the "sluggishness" of the system.

The time delay required to achieve adequate decorrelation is not easy to determine analytically since the error in spectral density of the power average signal is a function of (1) the number of signals, (2) the center frequency of the

measurement, (3) the bandwidth of the measurement, and (4) the relative magnitudes of the individual signals. For a given time delay, the error is smaller as items 1 through 4 increase, or conversely, larger time delays are required as they decrease. Clearly the low-frequency end of the test spectrum is governing since items 2, 3, and generally 4 will be minimum at this point.

The results of an empirical evaluation of the adequacy of the tape delay method are shown in Figs. 4-8 and 4-9. The data were obtained during identical random vibration tests of five missiles in which two accelerometer signals were power-averaged by using a tape delay of 200 msecs. The spectral density values were obtained using a 10-percent bandwidth analysis system [1] and 30-sec data sample lengths, i.e., 30-sec integration times. It is important to note that, except for the 200-msec delay, the same 30-sec sample of each signal was used in each case. The three curves of Fig. 4-8 represent the maximum, mean, and minimum values, in each analysis bandwidth, of the ratio of the spectral density of the power-averaged signal to the mean of the spectral densities of the two signals from which it was formed, i.e., the true power average. The three curves of Fig. 4-9 represent the maximum, mean, and minimum values in each analysis bandwidth of the ratio of the spectral densities of the two individual signals, illustrating that the curves of Fig. 4-8 were obtained over a wide range of relative magnitudes. Except for the first five bandwidths, from 19.6 to 34.7 Hz, the curves of Fig. 4-8 indicate the adequacy of the 200-msec delay. The 200-msec time delay represents delays of 4, 5, and 6 cycles in the first, third, and fifth channels, and delays of 4-1/2, 5-1/2, and 6-1/2 cycles in the second, fourth, and sixth cycles, respectively. The effects of reinforcing and canceling in the odd and even channels respectively due to inadequate time delay are evident.

The two curves of Fig. 4-10 are of the same ratio as those of Fig. 4-8 using the data from one of the five tests. One curve used a 200-msec delay as before, the second used a 400-msec delay. The improvement with longer delay is evident. Since random vibration equalizer/analyzer filters in this frequency range employ a bandwidth of at least 10 Hz, rather than the 2- to 4-Hz bandwidth in Figs. 4-8 through 4-10, it appears that 200 msec is generally adequate for power averaging. If greater delay is desired, either of two approaches can be used:

1. If one of the newer, wideband FM tape recorders is available, the tape speed may be reduced to 7.5 ips while maintaining the response to 2500 Hz. See Fig. 4-11.

2. If only the older type recorder is available, the double delay method diagrammed in Fig. 4-12 may be used. It is the less desirable alternative because two additional tape tracks, which might otherwise be used for data acquisition, must be committed to test control.

Commutation Method. The second averaging method is shown in Fig. 4-6, where the individual transducer signals are commutated at an appropriate switching rate so that the power-averaged signal consists of a series of time segments of the signals from each transducer. Each segment typically contains several cycles

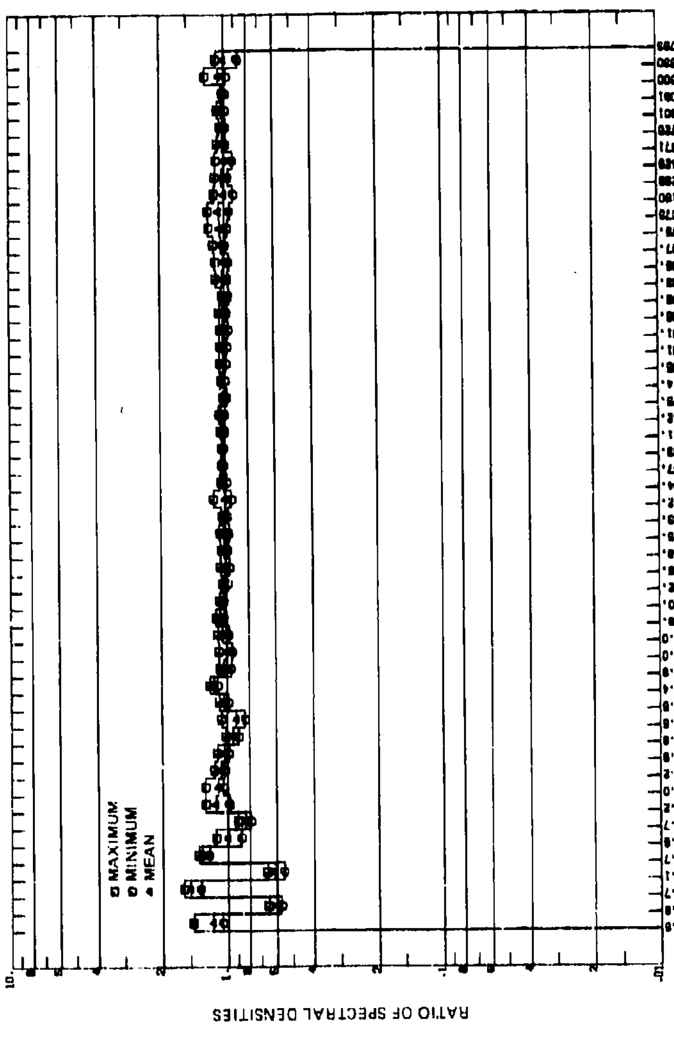

Fig. 4-8. Ratio of true power average spectral density to spectral density of TDM averager for five missile tests, using two accelerometers.

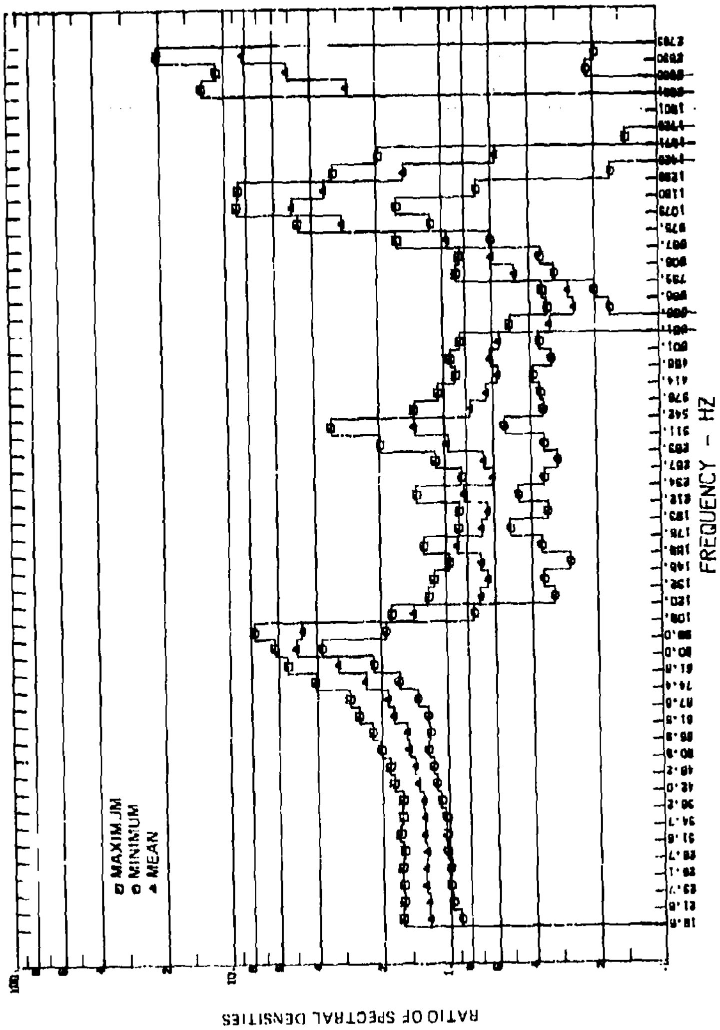

Fig. 4-9. Maximum, minimum, and mean of ratios of individual accelerometer signals forming power average in Fig. 4-8.

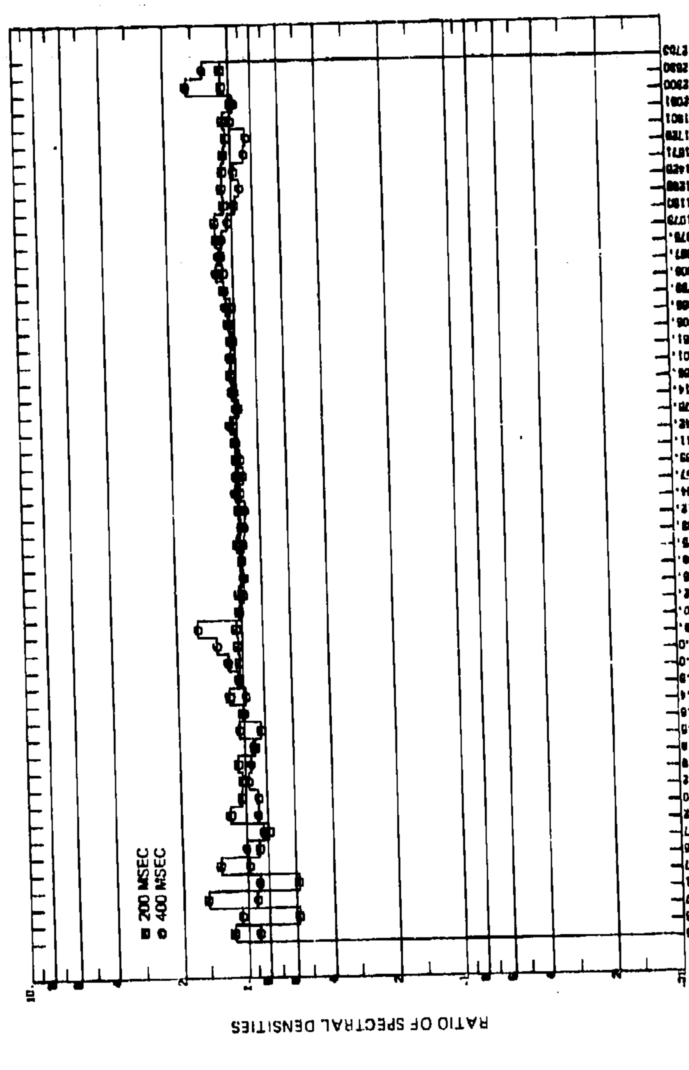

Fig. 4-10. Comparison of 200- and 400-msec time delays for power averaging.

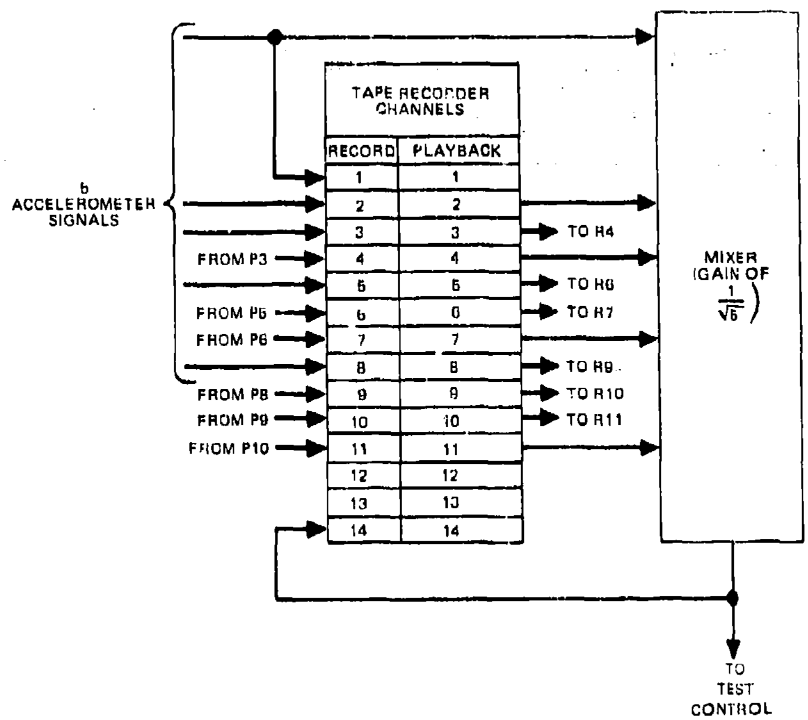

Fig. 4-11. Instantaneous averaging of random signals, tape delay method.

of the lowest frequency of the spectrum. Fig. 4-13 is a photograph of the output of the commutator (TDM). (One channel with zero signal input was used for illustrative purposes only. Actual use with a zero signal would bias the theoretical level by a factor of $(n-1)/n$ for n input channels.)

Performance of this type of power averaging is shown in Fig. 4-14, which is a plot of the ratio of the true power average of four accelerometer signals to the spectral density of the averager output signal, using 10-percent bandwidth analysis. The four signals were the same as those used for Fig. 2-3, thus indicating the range of the signals which were averaged. It can also be observed that sinusoidal waves do not average on a mean square basis as shown in the bandwidth straddling 60 Hz. A power signal synthesized in this manner is actually a nonstationary signal even though each segment is stationary. However, if the averaging time of the control system to which it is applied is sufficiently long to average over one or more scans of the commutator, the control system will react as if a stationary signal with the properties of the required power-averaged signal had

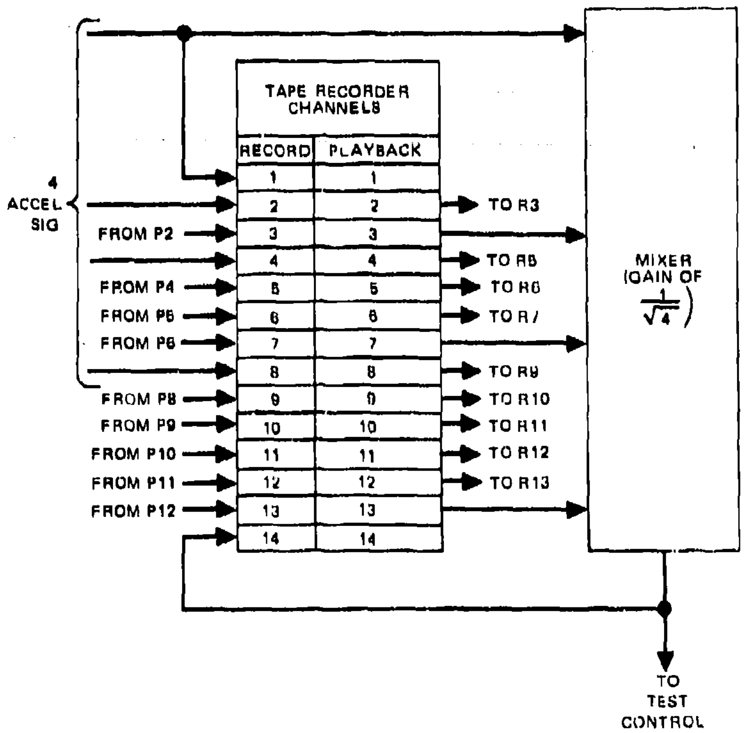

Fig. 4-12. Instantaneous averaging of random signals, tape delay method (400-msec delay at 15-ips tape speed).

been applied. There are two basic constraints on the application of this technique to random signals. First, if the sampling dwell time is made too small, the spectral density of the output deviates from the average of the input spectral densities. The degree of deviation is inversely proportional to the minimum bandwidth of peaks and notches in the spectral densities of the input signals. Second, if the dwell time approaches the averaging times of the analyzer channels, control instability, or "wow," results. See Refs. 87 and 88 for details.

A practical approach to the problem is to determine experimentally the dwell time which yields marginal control stability and then to reduce it by 10 to 20 percent for test control. Some early commercial versions of the device do not provide sufficient dwell adjustment range for the above. It is recommended that they be modified to permit settings somewhat greater than 100 msec.

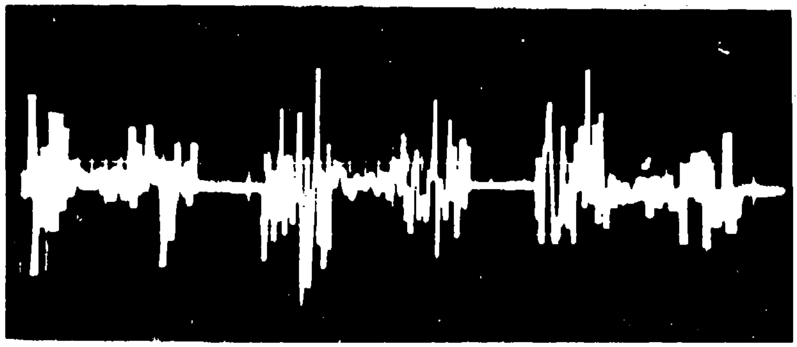

Fig. 4-13. Typical time division multiplexer (averager) output signal with three nonzero inputs and one zero input.

The use of the TDM for averaging poses potential problems in the performance of any type of random vibration test if the inputs have relative inverted polarities or widely differing amplitudes. The only effective remedial action possible is to correct the polarity inversion or to use the alternate tape delay method described above. With inverted inputs either of two types of errors can occur: (1) notches present in the inputs fail to appear in the output, or (2) peaks in the inputs are reduced in amplitude and spread over a wider bandwidth in the output. On the basis of limited empirical tests these effects appear to be pronounced at lower frequencies; however, it is suspected that the effects can occur at any frequency within the normal test range. Varying the TDM dwell time T has no perceptible effect on the first type of error but, with increasing T, the degree of frequency spreading is reduced somewhat for peaks. Figures 4-15 through 4-17 illustrate a typical notch error, they are 10-percent bandwidth analyses of actual test data where, because of test fixture configuration, one pair of control accelerometers was physically oriented 180° opposed to the other pair. The immutable operation of one of Murphy's laws also contrived alternate connections of one of each pair to TDM input channels 1 through 4. Figure 4-15 shows plots of the analysis of the TDM output and the computed average of the four input spectra (the latter are plotted in Fig. 4-16). Test control was, of course, based on the TDM output signal and resulted in almost 5-dB undertest in the equalizing channel centered at about 36 Hz (which was approximately the resonant frequency of an isolation-mounted element of the test item). As a final check, the taped individual signals were played back into the TDM with alternate channels reinverted and the TDM output analyzed. The result is plotted in Fig. 4-17 along with the computed average of the four input spectra.

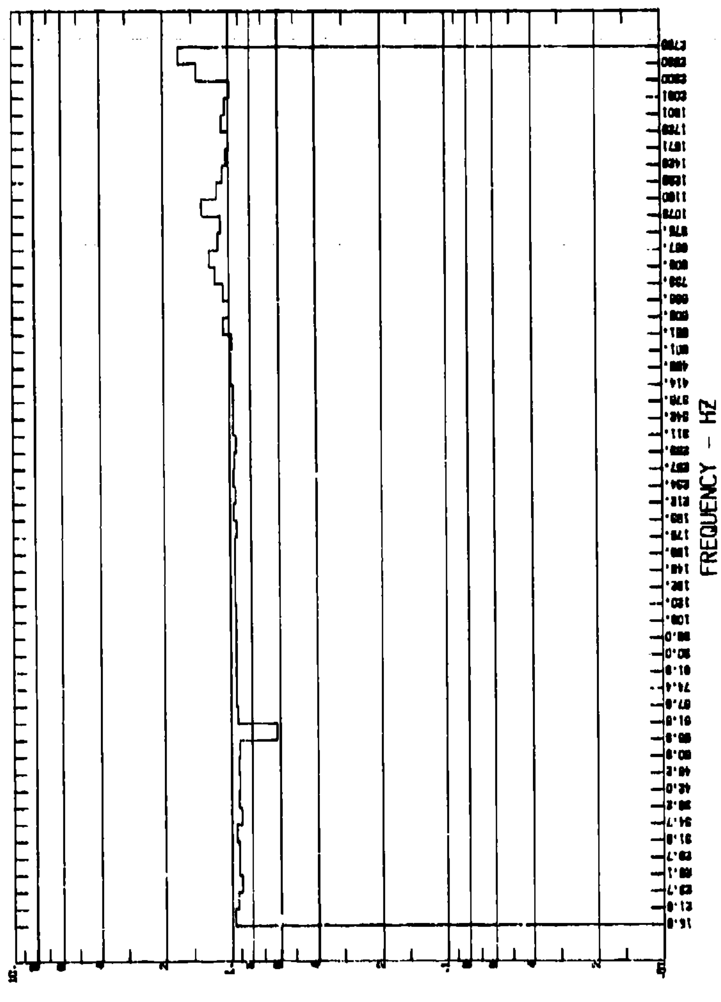

Fig. 4-14. Ratio of true power average spectral density to spectral density of TDM averager signal; four accelerometers used.

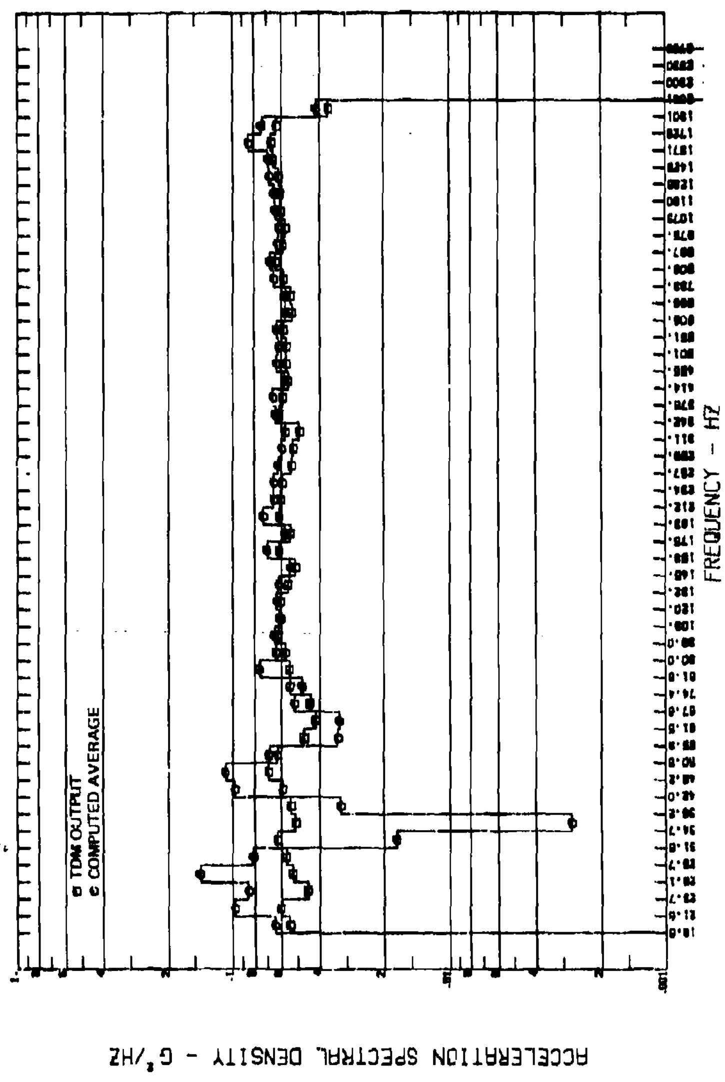

Fig. 4-15. TDM output and computed average of four inputs (adjacent channel polarities opposed).

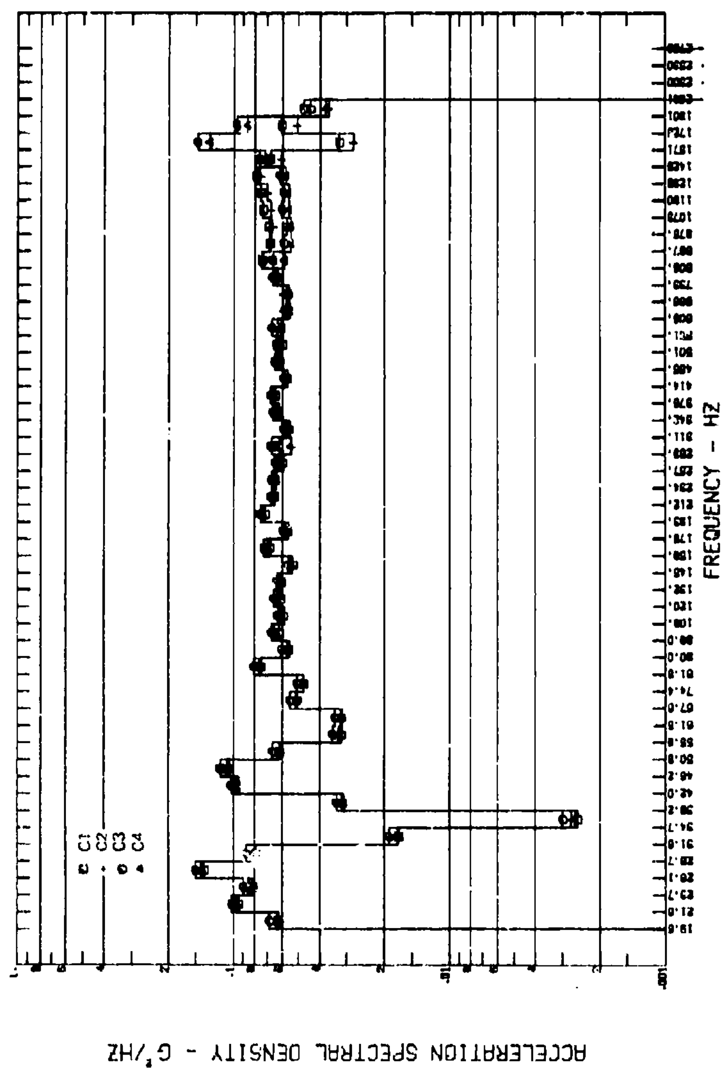

Fig. 4-16. Four TDM inputs of Fig. 4-15.

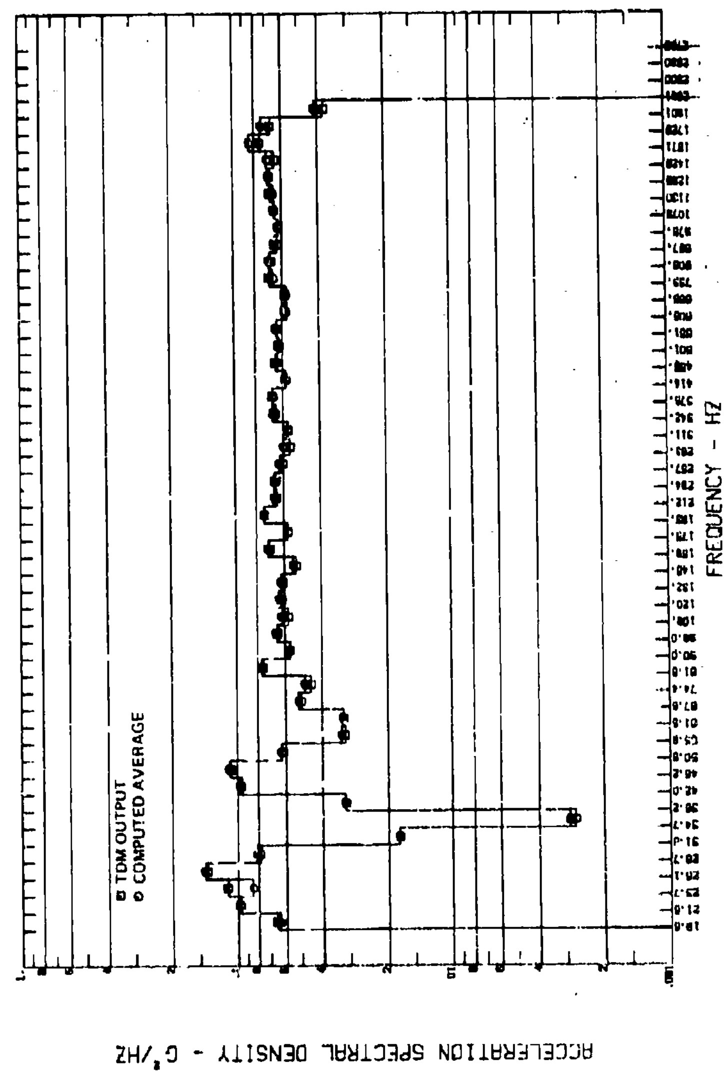

Fig. 4-17. TDM output (with input polarities corrected) and computed average of the inputs.

Relative amplitude differences between adjacent channels (without reversed signal polarities) also cause test errors. These errors are not extreme but the problem should be recognized, since they are systematic, they occur with only moderately large amplitude differences, and there is no obvious means for their correction during testing. The effect of error in the TDM output is to cause an increase in spectral content at lower frequencies followed by a gradual rolloff with increasing frequency. The effect is illustrated in Fig. 4-18 which shows plots of TDM output and the computed average of four inputs (the latter are shown in Fig. 4-19). The alternate inputs differed by a factor of 100 in spectral density and the output error ranged from about +1 dB near 33 Hz to nearly −3 dB at 2650 Hz. A dwell time of 100 msec (normal for random) was used; however, varying it between 50 and 200 msec had no significant effect on the results. Figures 4-20 and 4-21 similarly show the effect for inputs differing by a factor of ten. The approximate output error ranged from +0.6 dB at 44 Hz to −1.5 dB at 2650 Hz. The plot of Fig. 4-22, which shows analyses of the TDM output for four identical, nearly flat inputs and the computed average of the inputs, demonstrates that the TDM has no inherent roll-off effect.

Test Item and Facility Protection

In the performance of vibration tests, it is always necessary to provide protection against inadvertent overstressing of the test object and faults in the vibration equipment. Both forms of protection are required since a fault in the facility may cause test item damage and the test item may be overtested without exceeding facility performance limits. The adverse consequences of test item damage are obvious; the effects of facility damage in terms of costs and test delays are also an important factor.

Overstressing of the test object due to control factors can occur as a result of any of the following:

1. Instrumentation error; e.g., use of incorrect accelerometer sensitivity.
2. Operator error, either in manual control or in use of automatic control equipment.
3. Failure of automatic control equipment.
4. Loss of signal due to failure anywhere in the control transducer system.

The instrumentation error factor can be minimized by performing an independent sensitivity check of all control accelerometers prior to test by use of an optical wedge (Fig. 4-5) to measure displacement while applying sinusoidal vibration at a known frequency [80]. It is convenient to use 0.1-in. peak-to-peak motion which yields 1, 3, and 10 g's at 14.0, 24.2, and 44.2 Hz, respectively. The technique is particularly valuable where the check can be made at a fairly high level before installing the test item. However, even when the check must be made at a low level, gross errors can be detected.

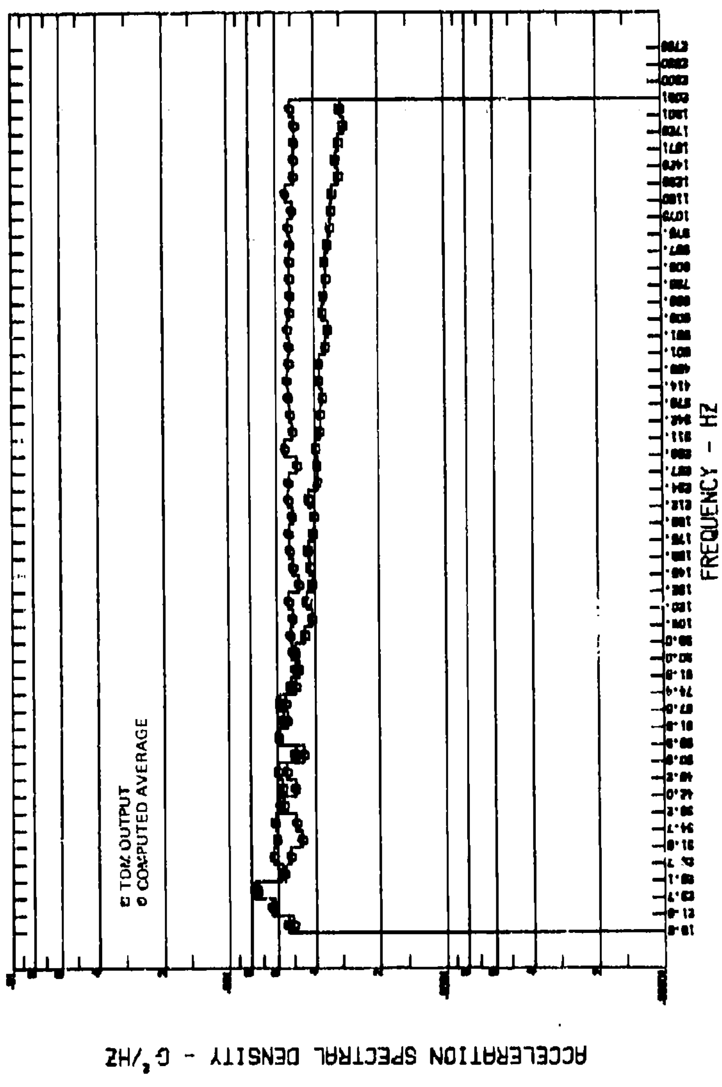

Fig. 4-18. TDM output and computed average of four inputs (adjacent inputs differing by a factor of 100 in power, 20 dB).

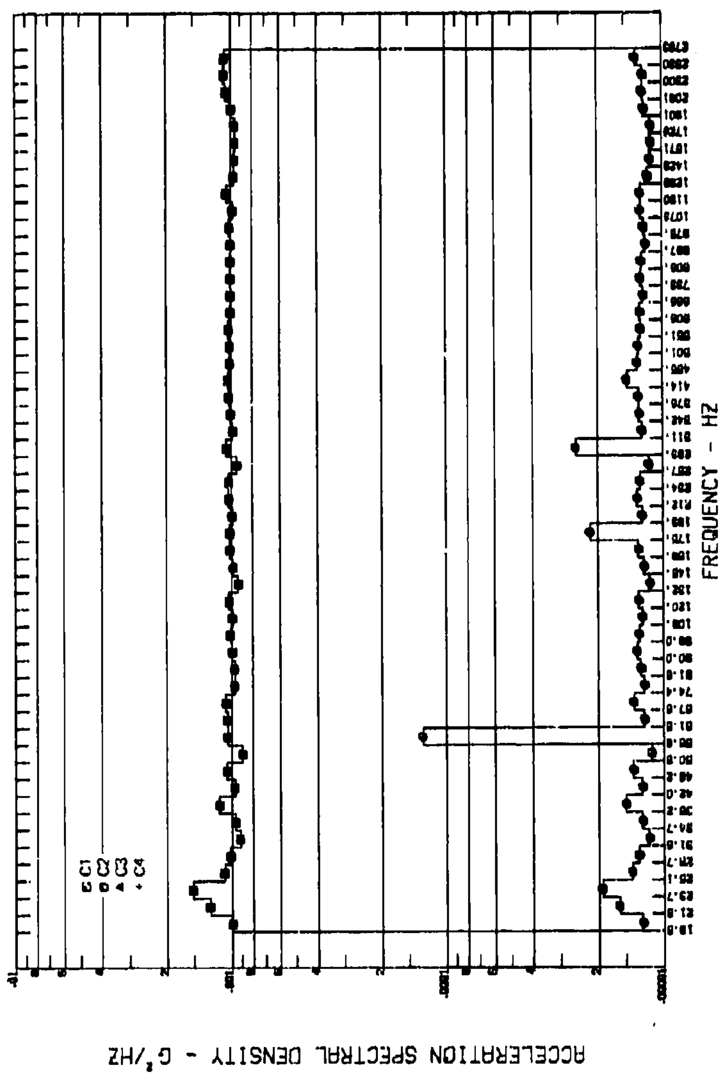

Fig. 4-19. Four TDM inputs of Fig. 4-18.

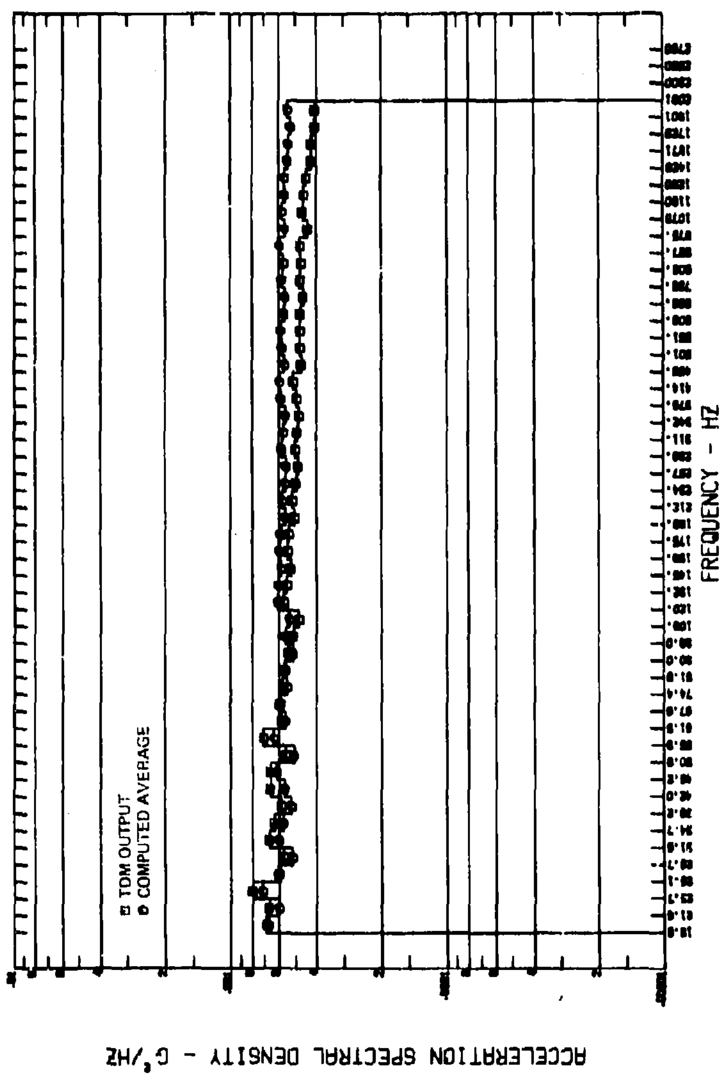

Fig. 4-20. TDM output and computed average of four inputs (adjacent inputs differing by a factor of 10 in power, 10 dB).

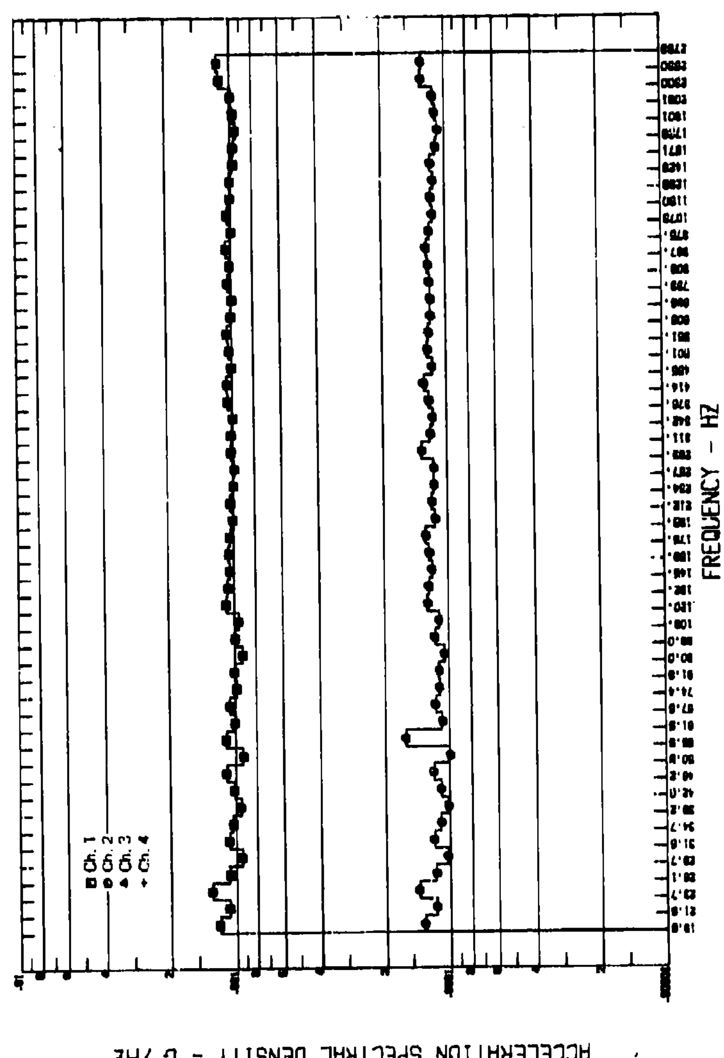

Fig. 4-21. Four inputs of Fig. 4-20.

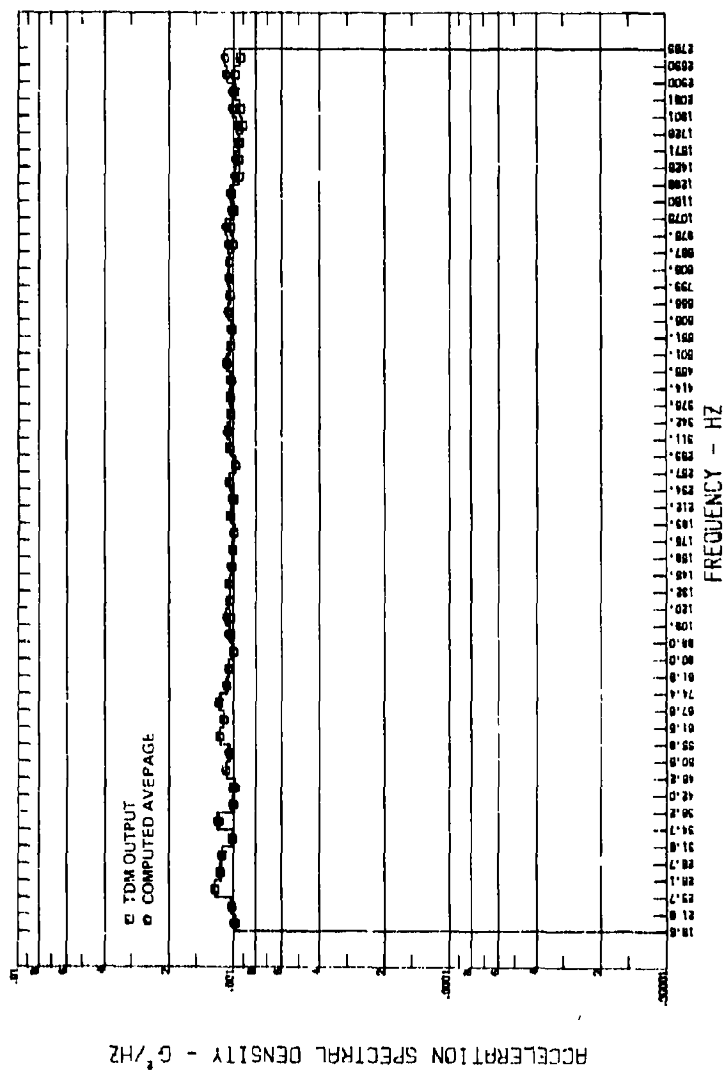

Fig. 4-22. TDM output and computed average of four identical inputs.

To prevent damage from the other potential faults, a combination of three basic techniques is recommended, particularly when the test item value is high.

1. Use of an acceleration threshold limiting device, or G-limiter [89]. The threshold is adjustable to a predetermined level and the device shorts the input to remove final stage power from the power amplifier whenever the control signal exceeds the preset level.

2. Use of a device (the "no-signal" detector) which monitors the control signal and performs the same protective functions when loss of signal is detected.

3. Use of a manual abort switch by the test engineer or other person able to detect abnormalities by visual and/or aural monitoring of test performance. The same protective functions can be initiated by the abort switch.

The electronic power amplifiers used to drive today's vibration exciters usually have incorporated in their design devices which sense over- and undervoltages, excess currents, overtemperatures, coolant flow, etc., and shut down operations to prevent or limit internal damage when abnormal conditions are detected. Most of the faults so protected against do not constitute a threat to the test article. However, because of its required large power handling capacity, the output stage does have a potential for serious damage to both the vibrator armature and test object. For example, the occurrence of a short between the grid and plate of an output tube could impress upon the armature an extreme accelerating force, resulting in catastrophic velocity and displacement. This could occur despite operation of overcurrent relays and removal of power because of the large amount of energy stored in the filter of the high voltage supply. From the early days of random vibration testing, before this potential was recognized, one of the authors has an all-too-vivid memory of an occurrence of just such a fault which left an armature dangling by one flexure in midair above the shaker case. It has become common practice to use what is called an armature protector to guard against potentially catastrophic faults [90]. The devices rely on the very fast operation of gas switching tubes (thyratrons or ignitrons, popularly called crowbar tubes for this use) which usually perform two functions: (1) shorting the high voltage supply to ground to remove the drive energy source and (2) shorting the input windings of the output transformer to provide electrodynamic braking of the armature from the back-emf generated by it as it moves through the shaker magnetic field. The triggering function may be derived by sensing excessive velocity, acceleration, return current, or displacement, although the latter is of doubtful value since its occurrence usually will be too late in the chain of events to permit effective remedial action.

The need for use of armature protection unfortunately creates another problem with respect to the safety of the test article. Operation of the device may result in overstressing of the test object. This problem can be alleviated by the application of an electrodynamic braking technique described by Cook [91]. However, the potential problem reinforces the need for the test item protective

techniques described earlier, in order to prevent externally imposed conditions which might trigger the armature protector.

Test item protective techniques, if applied properly, must complicate and thus increase test costs. This follows from the need for some form of confidence check of satisfactory performance of each function just prior to test; otherwise, one may be relying on nonexistent protection. For this reason, the requirement for and the degree of protection should be evaluated carefully with respect to the value of the test item.

Equipment Calibration and Alignment

Regardless of all other precautions taken, test performance can be no better than the quality of the instrumentation used. Standard monitoring equipment such as voltmeters, counters, etc., are usually subject to periodic calibration and certification checks; use of such equipment beyond the calibration period should be avoided. Transducer calibration requires specialized equipment and techniques not available in many test laboratories [79,92-95]. Gross changes in sensitivity between calibration and test use will be detected if the optical check recommended in the previous section has been made. Alignment instructions should be followed carefully for specialized equipment such as tracking filters and random vibration equalizer/analyzer systems.

CHAPTER 5
TEST PERFORMANCE AND CONTROL

With the exception of an introductory section on control techniques and equipment functions, the material in this chapter is arranged by types of tests. The reader is referred to Chapter 4 for test performance considerations which are generally applicable to all tests.

5.1 Programming and Control

Control Techniques

Selection of an appropriate method for programming and control of vibration levels fo- any given test will depend upon a combination of factors, examples of which a e

1. 'he simulation requirements discussed in previous chapters.

2. The purpose of the test.

3. Number of control transducers required (often, but not always, a function of test object size).

4. Data requirements. For example, for a simple resonance search or transmissibility measurement, precision of control may be less important than data accuracy.

5. Availability of control equipment. Probably more compromises of choice stem from this than any other factor.

The technique chosen may range from simple manual control based on a single transducer to complex automatic control based on multiple transducers and the use of signal selection or averaging. Detailed factors to be considered in the selection of methods of control for various forms of vibration and combinations of equipment are presented in later sections of this chapter dealing with each form of test.

Basic Equipment Functions

For any form of vibration test, there may be defined three basic categories of equipment functions external to, and used to control, the power amplifier-shaker combination (see Fig. 5-1). They are

1. Control (A). This includes monitoring functions and gain controls (both manual and automatic) as well as the excitation source.

2. Control signal generation (B). Included are transducers and the connected equipment required to convert their output signals to a form compatible with the control equipment.

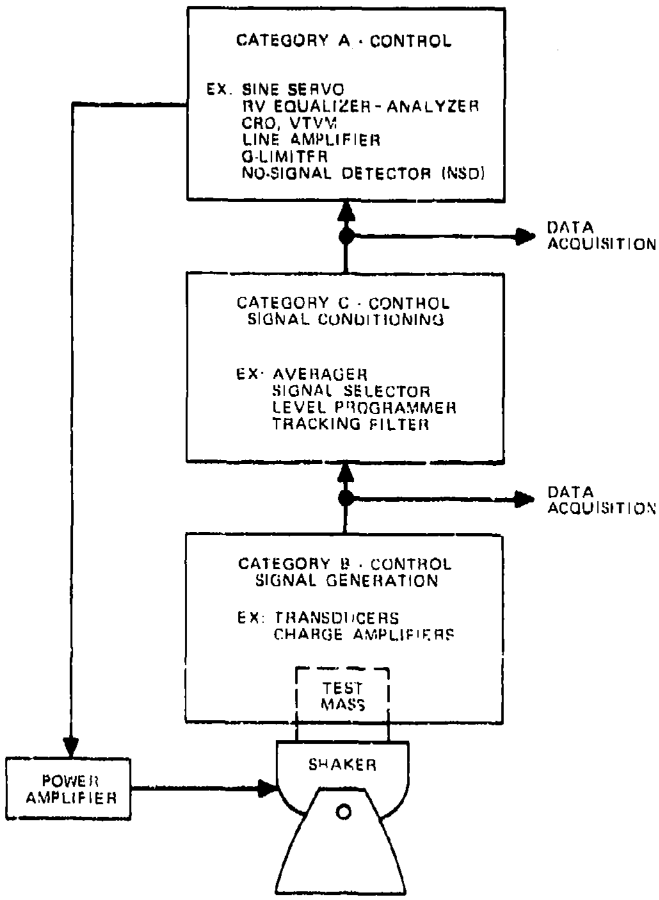

Fig. 5-1. Basic categories of equipment functions used for test control.

3. Control signal conditioning (C). This category includes functions such as signal selection, averaging, filtering, and level programming.

In the discussion of specific forms of tests in following sections there will be found repeated references to servo time constants. For the reader who may be unfamiliar with servo (automatic control) equipment, a brief explanation is in order. In a servo, the vibration excitation signal is fed through a variable-gain amplifier. The amplifier gain is controlled by an error signal which is generated by comparing the detected and smoothed control (feedback) signal to an adjustable dc reference voltage. The rapidity with which the servo can respond to, and correct for, changes in the control signal is determined by the servo time constant. This is a composite of time delays in the system, but the chief contributor is the detector averaging circuit. Both manual and automatic changes in servo rate are effected by varying the detector time constant. There are major differences between servo functions for sinusoidal and random testing.

1. Sinusoidal. The manual servo rate adjustment (usually designated as *compression speed*) is an operator control but the detector averaging time also is varied automatically as a function of frequency while sweeping.

2. Random. Both servo rate-determining and detector averaging times are fixed for each frequency channel. The "high" and "low" damping modes, with which some readers may be familiar, affect only the readout meters and not the feedback signals.

5.2 Sinusoidal Tests

The material in this section is restricted to what is commonly called the simple sinusoidal vibration test (which often turns out to be not nearly as simple as we would like), with the excitation derived from a single oscillator. It is convenient to identify three general types: (1) swept, (2) resonance search, and (3) dwell. A distinction is made between the first two because equipment and techniques used can differ widely.

Swept

Modern practice revolves around the use of cycling oscillators, electronic servo-controllers, and other automatic programming equipment. However, it is worth noting that even the most complicated test can be performed by substituting operator skill for one or more of these functions and breaking up the test into partial sweeps more amenable to manual control. The major virtue of the use of automatic equipment lies in reduced test time and the nominal capacity for precision of control and replication of test parameters from sweep to sweep. The term *nominal* is used advisedly, since the potential advantages of such equipment are not always realized in practice.

The swept sine test may take any of several forms. These range from constant acceleration vs frequency to complex schedules of displacement, velocity, and

acceleration vs frequency. Each may be complicated further by a requirement for filtering the feedback signal [96] in order to control the level of the fundamental where resonances create distortion; by requiring that control be based upon the alternative selection of one of several transducer signals, depending upon their relative amplitudes; or by requiring the use of the averaged output of several transducers as the control signal.

Swept, Unfiltered. The simplest form requires only maintenance of constant acceleration vs frequency. If servocontrol is used (Fig. 5-2), the servo time constant adjustment must be compatible with sweep rate selection (refer to pp. 57-60 for selection criteria). The optimum time constant cannot be predetermined since it will depend on both sweep rate and the response characteristics of the test mass and the vibration system. It should be noted that the

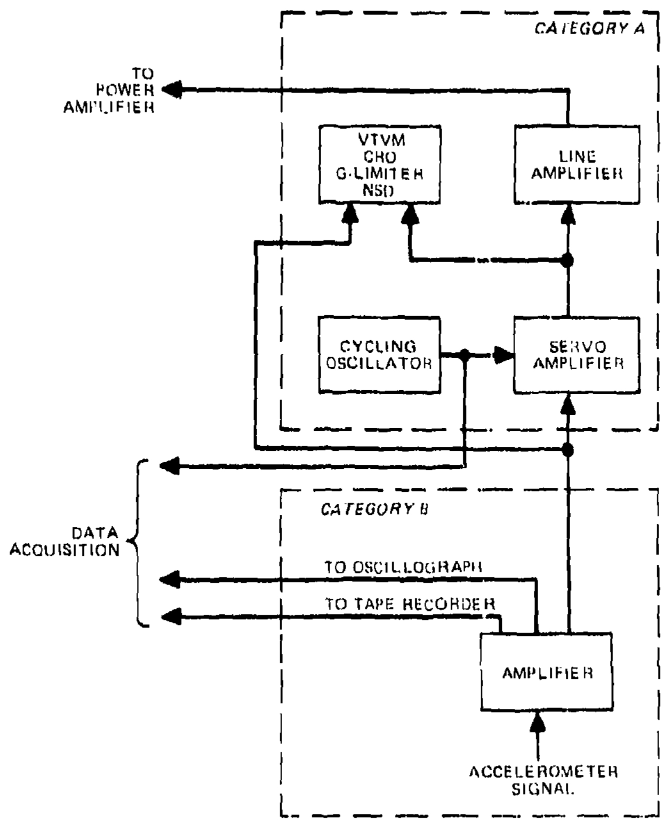

Fig. 5-2. Functional diagram for swept sinusoidal test, unfiltered.

required time constant is an inverse function of sweep rate and sharpness of resonances encountered, and that the performance characteristics of the available servo system may require a compromise of sweep rate selection (pp. 57-60).

Below some relatively low frequency, it is often necessary to make transition of control from constant acceleration to constant displacement. This is, of course, a form of level programming but is not treated as a category C function because for many years sine servocontrol systems have provided a built-in capability for performing the function. It is commonly achieved by using a dc analog of frequency to switch control from the acceleration signal to a displacement signal. The latter is usually derived through double integration of the acceleration signal but may, in most servos, be generated alternatively by single integration of the output of a velocity transducer. In some later servos, control switching is effected by use of a signal comparator which transfers control to the larger of the two signals.

Swept, Filtered. The functional diagram for this test is shown in Fig. 5-3. The introduction of the tracking filter into the feedback loop complicates the selection of sweep rate and servo time constant. This is because there is a delay between the time of change of input signal amplitude and the time the filter output responds to the change. The amount of delay is an inverse function of filter bandwidth. Since this is an added delay in the servo feedback loop, for a given sweep rate and filter bandwidth there is a limited permissible range of adjustment for the servo time constant which will result in good test performance. Ideally, the process of defining test requirements would take account of this factor. Since this requires a fairly detailed knowledge of the characteristics of the actual equipment to be used, it is rarely possible to do more than provide some latitude in test requirements which will permit effecting reasonable solutions to the inevitable problems that will arise. The problems can be minimized, however, by specifying the lowest sweep rate and widest filter bandwidth compatible with test objectives and cost limitations.

Level Programming. This refers to the fairly common practice of specifying a sweep where, at intermediate frequency points, a change in vibration amplitude is required. Various combinations of displacement and acceleration vs frequency may be specified; rarely, there may be a requirement for control of velocity. In the latter case, it is usually necessary to perform the sweep piecewise, since automatic equipment to effect the required control transition does not appear to be available commerically. Where the required transitions are limited to displacement and acceleration, it is occasionally necessary to break up the sweep into combinations of displacement followed by sequential acceleration level changes because many available servo-controllers can handle only a displacement schedule followed by acceleration schedules. A typical curve defining vibration amplitude as a function of frequency is depicted in Fig. 5-4.

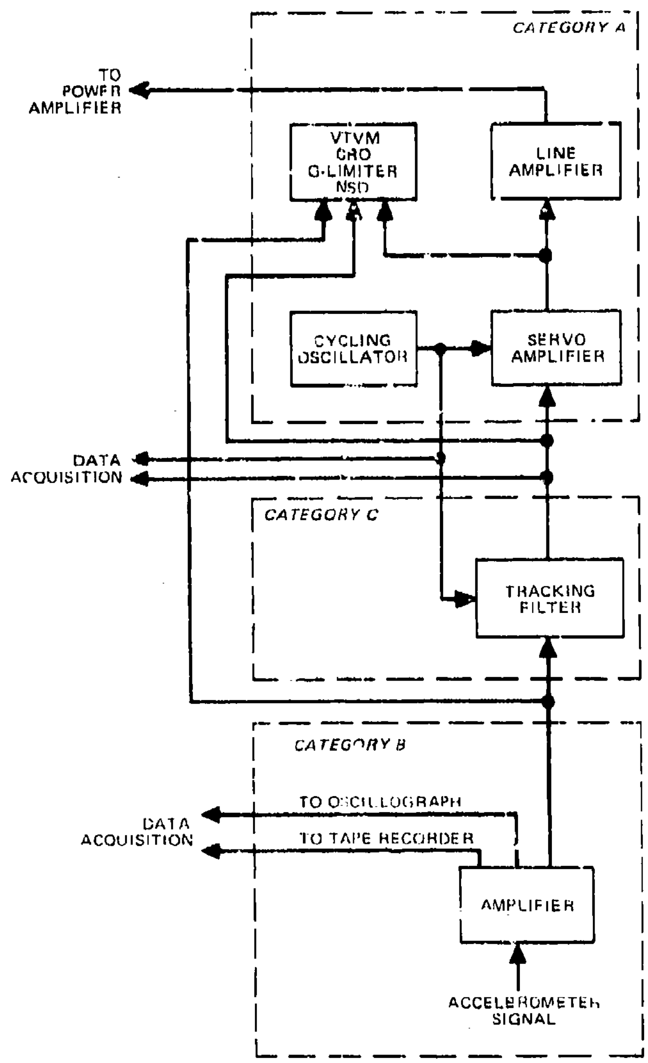

Fig. 5-3. Functional diagram for swept sinusoidal test, filtered.

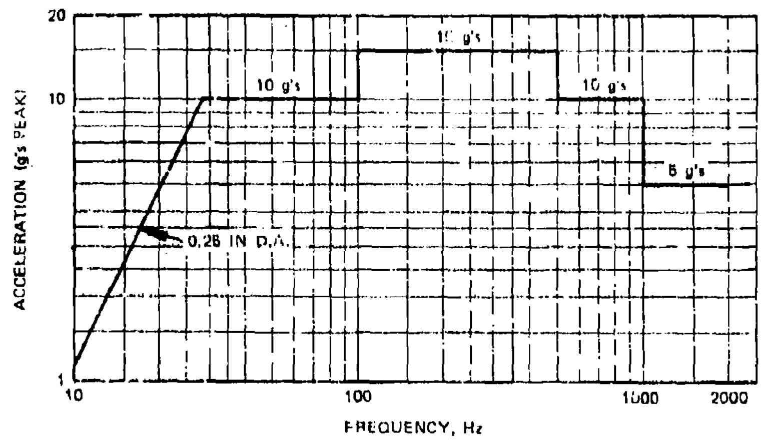

Fig. 5-4. Typical test requirements, swept sinusoidal with level programming.

The application of level programming is shown schematically in Fig. 5-5 for an unfiltered sweep and in Fig. 5-6 for a filtered sweep. The level programmer is a device which contains several switchable channels, the gains of which can be varied. The frequency at which switching occurs is set by adjusting a dc threshold voltage in each channel to correspond to the dc analog of the frequency at which each level transition is desired. There is commercial equipment available which allows the switching frequencies to be preprogrammed with a conductive ink chart on a curve follower.

For unfiltered low-level sweeps, there may be a problem of spurious triggering of the G-limiter due to channel switching transients, which in some level programmers are uncomfortably high. However, judicious low-pass filtering of the G-limiter input minimizes the problem. For filtered sweeps, an additional constraint is imposed on the selection of sweep rate, servo time constant, and filter bandwidth. The degree of constraint depends on the magnitude of level change, the direction of change and switching speed, and their impact on the performance of the specific tracking filter and servo combination used. Hence, this factor can only be noted as a potential problem requiring empirical solution during test performance.

Signal Selection. The application of signal selection is diagrammed in Fig. 5-7 for the unfiltered sweep. Most commercial versions of the signal selector contain a combination of switching logic and adjustable threshold levels in each channel which cause transfer of control to whichever acceleration signal has, at the moment, risen to the preset level for its channel. A few such devices also provide

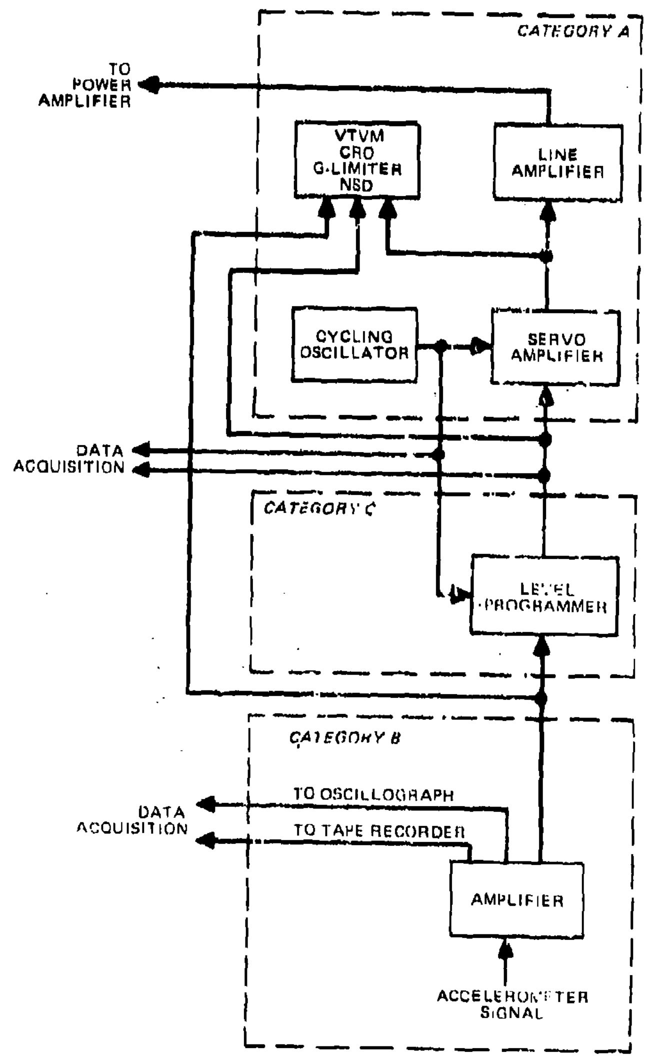

Fig. 5-5. Functional diagram for swept sinusoidal test, unfiltered, with level programming.

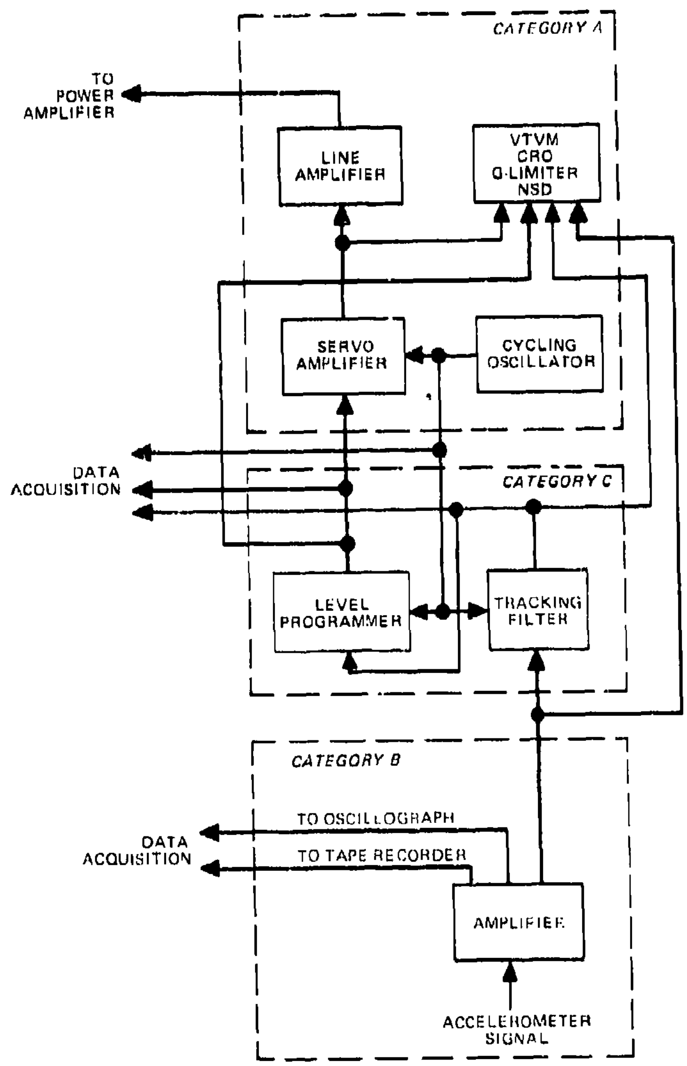

Fig. 5-6. Functional diagram for swept sinusoidal test, filtered, with level programming.

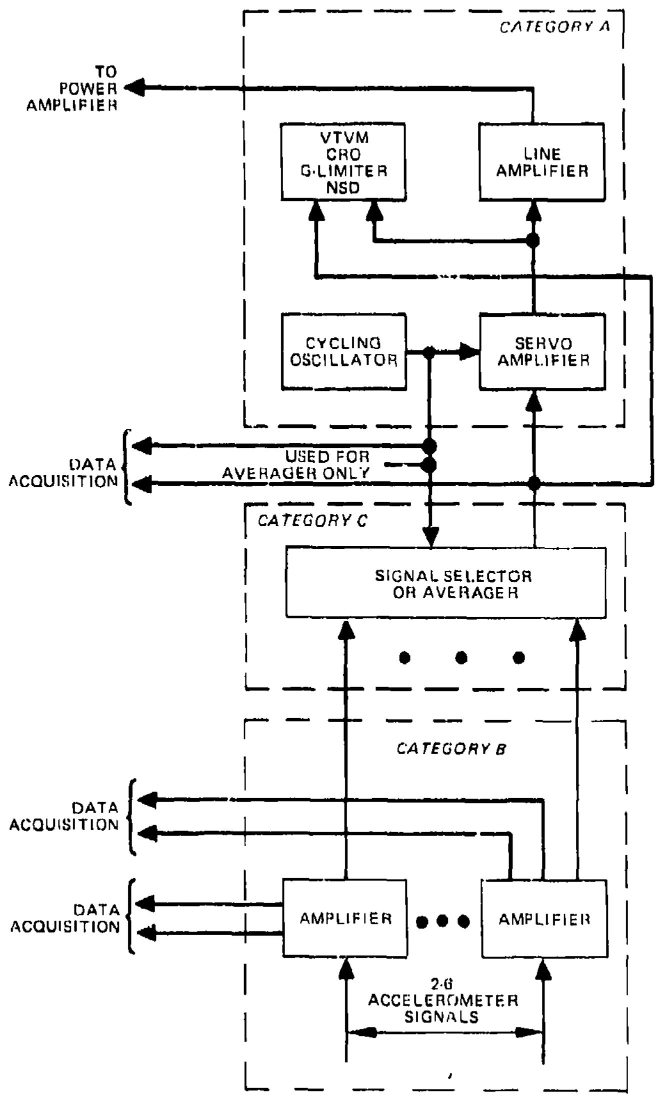

Fig. 5-7. Functional diagram for swept sinusoidal test, unfiltered, with signal selection or averaging.

an alternative operating mode in which channel selection is based upon minimum rather than maximum signal levels. This is necessary in order to meet one of the requirements of MIL-STD-810B, Method 514, Para. 5.4. Switching transients may cause spurious triggering of the G-limiter but, here again, the problem can be minimized by adequate low-pass filtering of its input. However, if selector switching is not synchronized well enough to limit the duration of signal dropouts to a value somewhat less than the loss-of-signal detector (NSD) averaging delay or the servo time constant (whichever is smaller), either the test will be aborted by the NSD or control instability will result.

For the filtered sweep with signal selection (Fig. 5-8), in addition to the potential problems cited above, the sweep rate/filter bandwidth limitations discussed on p. 143 must be considered.

Averaging. A schematic representation of test control employing averaging is shown in Fig. 5-7 for an unfiltered sweep. Early versions of the averager required detection of each acceleration signal before averaging in order to avoid phase cancellation effects. Such devices could be used only for very slow sweeps because of the time required for the detection process. The present-day averager avoids this problem by commutating the acceleration signals and averaging the resulting composite output consisting of sequential time samples of each signal. For the unfiltered sweep, the rate of commutation of the signal is synchronized with the sweep frequency so that each successive sample contains an equal number of periods (usually one). Thus, regardless of relative signal amplitudes and phase, the detected and smoothed output is proportional to the true average [87].

When this technique is applied to the filtered sweep (Fig. 5-8), operational compromises are required due to the effects of filter bandwidth and commutation dwell time on servo time constant optimization. Sweep rate selection, of course, is affected also. Limits are imposed on the ratio of servo time constant to dwell time, on the product of filter bandwidth and dwell time, and on the sweep rate which may be used; see Ref. 87 and Section 4.3, page 117 for details. Some latitude in sweep rate selection can be gained by switching filter bandwidth and time constant at higher frequencies, but equipment setup and operation are complicated considerably. If accelerometer polarities are opposed, special precautions are required (see Section 4.3, page 126). The cited problems can be avoided largely by using a tracking filter in each signal channel preceding the averager, but such equipment is expensive and test setup complexity is not reduced significantly.

Level Programming with Selection or Averaging. Test control employing level programming with signal selection or averaging is diagrammed in Fig. 5-9 for the unfiltered sweep. Except for increased test setup complexity, potential problems remain essentially the same as those already cited for application of the individual functions; i.e., possible adverse consequences of switching transients.

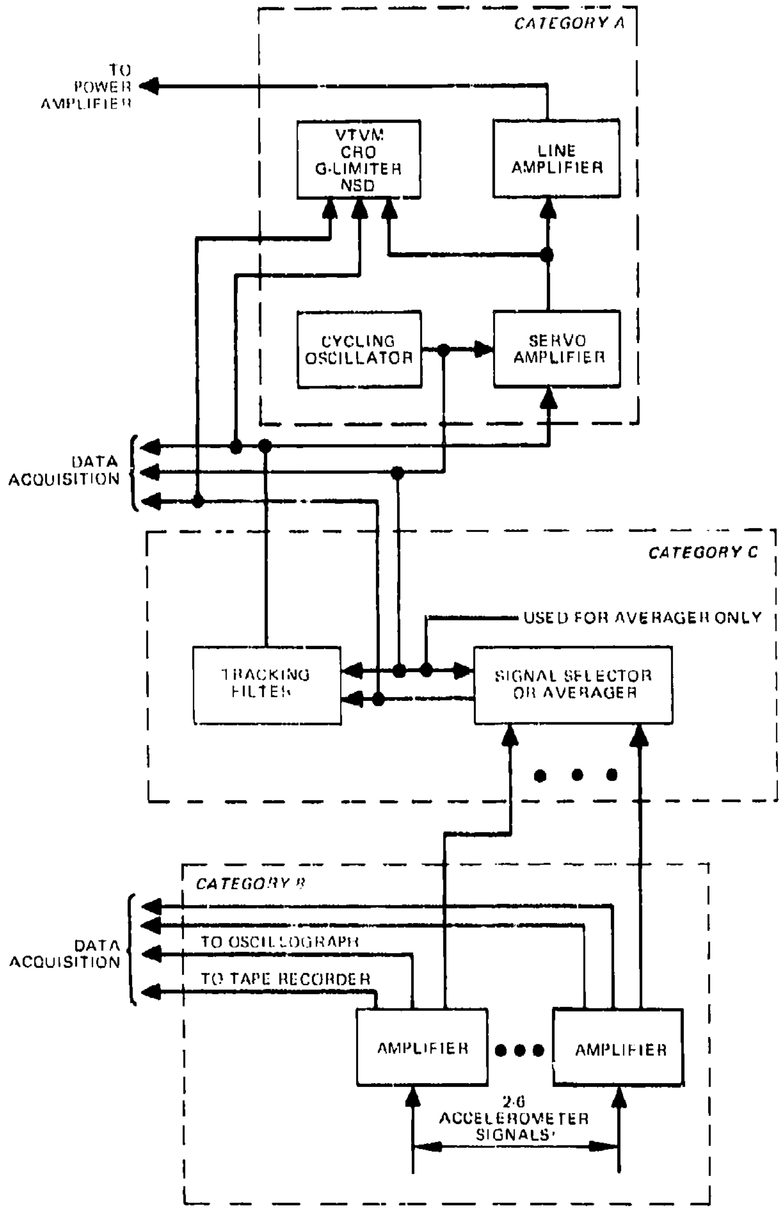

Fig. 5-8. Functional diagram for swept sinusoidal test, filtered, with signal selection or averaging.

For the filtered sweep with level programming and signal selection (Fig. 5-10), the increased test setup complexity becomes a significant factor. Equipment interaction problems are about the same as those noted before for the individual applications. However, for the filtered sweep with level programming and averaging (Fig. 5-10), the problems of selecting sweep rate, servo time constant, filter bandwidth, and averager dwell time are compounded. The interactions between averager, tracking filter, level programmer, and servo are too complex to be predefined. Since it is likely to be extraordinarily difficult to perform, the decision to specify this test should be based upon critical examination of all possible alternatives.

Resonance Search

As has been noted in Section 3.2, the resonance search test is primarily a preliminary to the performance of a resonance dwell test but is commonly used also for determining transmissibilities. It is convenient to use servocontrol to maintain the constant acceleration input to the test item attachment point chosen as a reference. The usual practice, unfortunately, is to employ the unfiltered sweep which may often yield misleading results. First, at many resonances there is likely to be considerable harmonic distortion in the control signal; since the servo will operate on the composite signal, the input at the control frequency may be considerably lower than nominal. Second, a resonance will respond differently to a rapidly varying input amplitude at the excitation frequency than it would if the input were kept constant. Third, if, as is commonly the case, estimation of peak responses is based on reading oscillographic traces, a resonance may be defined as significant at some frequency, where a major portion of the response is at some multiple of that frequency. The response when the sweep reaches the latter frequency is likely also to be defined as significant. Then subsequent performance of the dwell test at the two frequencies will effectively dwell at the same resonance twice. Consequently, the use of the filtered sweep (see Fig. 5-3) is recommended for resonance search testing. Response data must be tape recorded for subsequent playback through a tracking filter for the generation of oscillographic records for use in evaluating resonant responses. Refer to the discussion on p. 143 for test performance factors to be considered.

Occasionally the use of response accelerometers to detect resonances must be supplemented by another technique because the test item cannot be instrumented adequately. If the structure of the item can be monitored visually, the stroboscope is useful below 300 to 400 Hz. Pinpointing the resonant frequencies requires disabling the oscillator sweep drive and manual vernier adjustment of the frequency to determine the point of maximum response. Slow-motion strobscopic photography [76] may be used as an additional diagnostic tool. Hand-held probes are sometimes used but, since readings obtained are variable

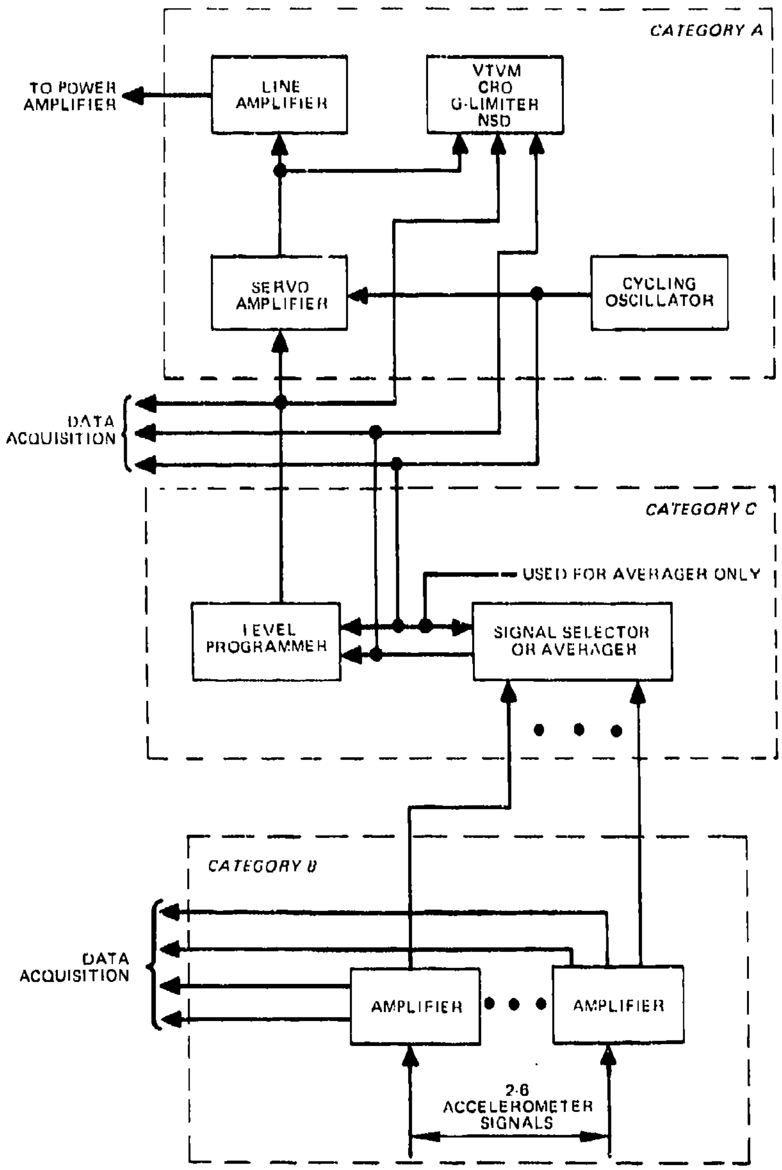

Fig. 5-9. Functional diagram for swept sinusoidal test, unfiltered, with level programming and signal selection or averaging.

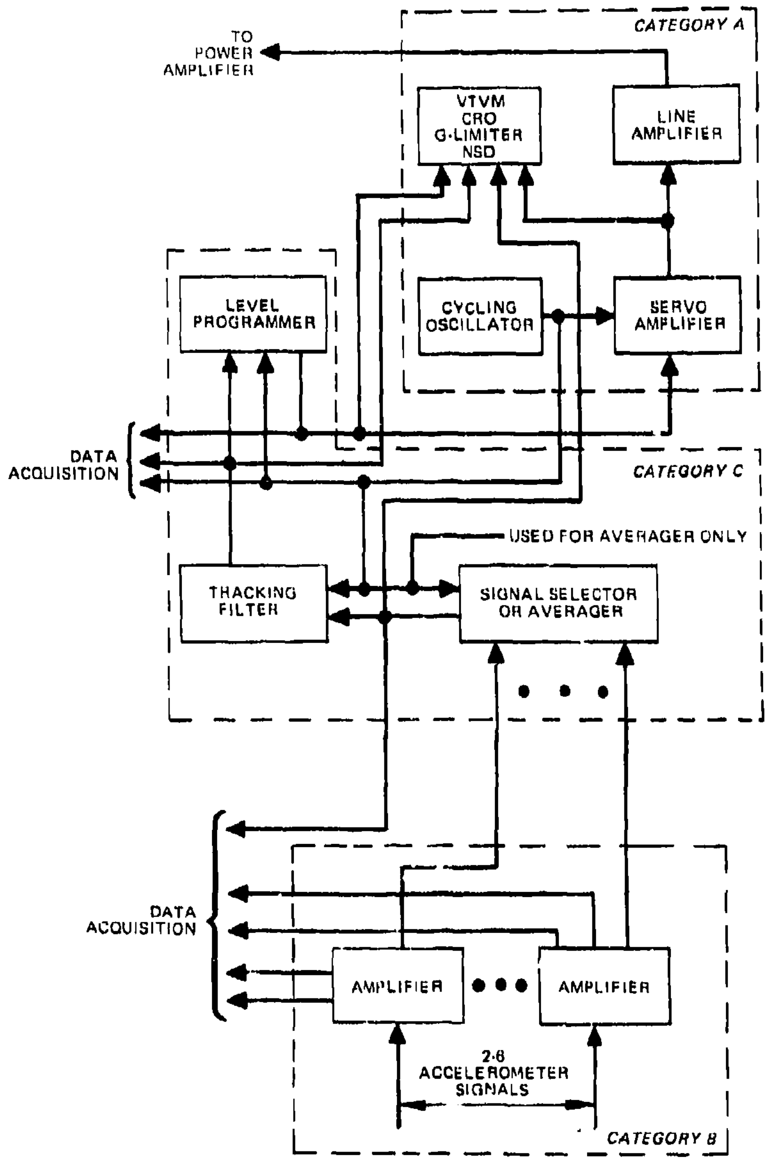

Fig. 5-10. Functional diagram for swept sinusoidal test, filtered, with level programming and signal selection or averaging.

and they are difficult to apply without loading the structure under examination, the results are always open to some suspicion.

As has been noted earlier, in the measurement of transmissibilities, precision of control of the input is less important than data accuracy. Thus, it is not strictly necessary to use the filtered sweep for this purpose if the control and response data are recorded carefully on magnetic tape. However, the overall time for test performance and data analysis can be reduced if a filtered sweep is used. For details refer to Section 6.3.

Resonance Dwell

The performance of this test is relatively straightforward. Since considerable dwell time usually is required at each resonance, it is desirable to use servocontrol to insure maintenance of the desired input level regardless of attention span deficiencies of the operator. The decision as to whether the control signal should be filtered or unfiltered is easily made; if filtering is used in the resonance search, it should be used also for the dwell test and vice versa. However, the effect of filtering can be approximated if the unfiltered input level has been logged during the filtered sweep search and is duplicated in the dwell test.

During the dwell accumulated fatigue effects or the loosening of structure may cause a gradual change in the resonant frequency. This factor may be taken into account by periodic adjustment of the frequency to maximize the response. Such adjustment may be effected manually; as an alternative, commercial equipment is available which tracks the resonant frequency automatically.

5.3 Sinusoidal and Random Tests

These test requirements were first evolved before the development of the tracking filter and the automatic equalizer. Consequently, early test techniques involved separate preliminary tests for sinusoidal and random motion and the use of a two-track magnetic tape recorder and electronic mixer for programming the actual test. With the advent of tracking filters and automatic equalizers, test performance became simpler and much less time consuming.

Tape Programming

This method requires a three-step procedure illustrated schematically in Fig. 5-11. Using servo control and the specified sweep rate, a preliminary sweep is run while recording on magnetic tape the resulting modified input to the power amplifier. A separate preliminary test is then performed to equalize for the random portion; when satisfactory results are achieved, the equalized input to the power amplifier is recorded on a second track of the tape recorder for the required test duration (paralleling the previously recorded sine sweep). For the actual test, the tape recordings are played back through a mixer into the power

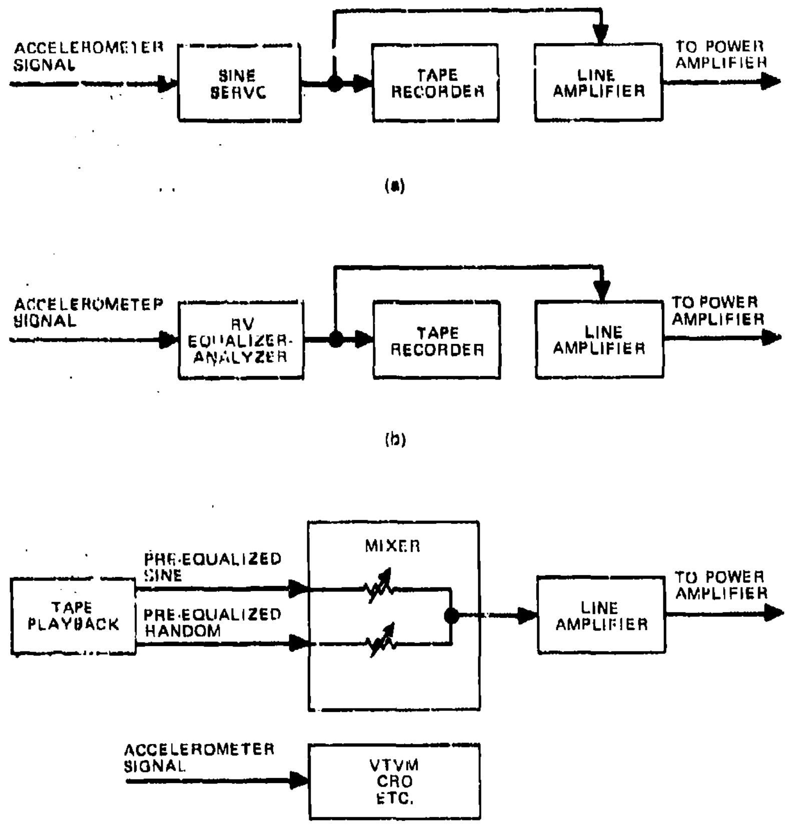

Fig. 5-11. Sinusoidal plus random test, tape programming method: (a) prerecording equalized sine sweep excitation, (b) prerecording equalized random excitation, and (c) test performance.

amplifier. Separate gain controls are used in each mixer channel to permit adjustment of the relative levels of the sine and random outputs.

There are two major disadvantages to this test procedure. First, in order to avoid pretest damage to the test item, it is desirable to perform the preliminary tests using a second test item or a very good dynamic model, so that equalizing may be done at actual test levels. However, the mechanical fit of the test item and fixture will usually vary enough from model to model (because of tolerances

and variation in fastener torquing) so that the test mass response in an actual test will differ from that obtained during preliminary tests. If the second test item is not available, preliminary equalizing must be done at a reduced level. The actual test levels are then also likely to vary unpredictably from those desired. The degree of uncertainty depends upon how nonlinear the test mass is. Second, the procedure is time consuming and requires uncommon expertise on the part of the operator.

It is possible to program the sinusoidal level and to use averaging for both sinusoidal and random. The three-step procedure is shown schematically in Fig. 5-12. The constraints noted in Section 5.2 are applicable.

Tracking Filter Method

The tracking filter used for the sinusoidal-random test provides two outputs. One is the normal narrowband filtered output which is used as the feedback signal to the servo. The second output is derived from circuitry which provides narrowband rejection at the sinusoidal frequency; it is used as the feedback signal to the random equalizer. Fig. 5-13 diagrams this test setup. It is desirable to use a filter as narrow as possible and a correspondingly long sweep time to insure sinusoidal control at the required level. This results from the fact that servocontrol is based upon a composite in the filter output of the sinusoidal signal plus the random signal passed by the filter. As filter bandwidth and undesired random signal increase, servo action reduces the sinusoidal amplitude, resulting in a degree of undertest for the sinusoidal portion. The effect may be compensated for by artificially increasing the sinusoidal control level, but explaining the resulting test data records is a particularly frustrating task.

Level programming may be applied to the sinusoidal portion and averaging may be applied to both, providing that the precautions in Section 4.3 (pp. 116-131) are observed. Fig. 5-14 illustrates this schematically. A constraint is added to those cited in Section 5.2 (p. 143) if averaging is used because optimum sampling dwell time considerations are different for sinusoidal and random signals. See Ref. 87 for further information.

5.4 Random Vibration Tests

Random Vibration Averaging

Where the control of a random vibration test is to be based on the average of multiple accelerometers, two methods are available. The signal commutating, or multiplexing, device or the magnetic tape delay technique discussed in Section 4.3 (pp. 118-131) may be used. However, if the former is used, special precautions must be observed, as described in the cited section.

Broadband Random Tests

The first known faltering attempts to perform random vibration tests were made in 1954. Early programming techniques evolved by a few pioneering

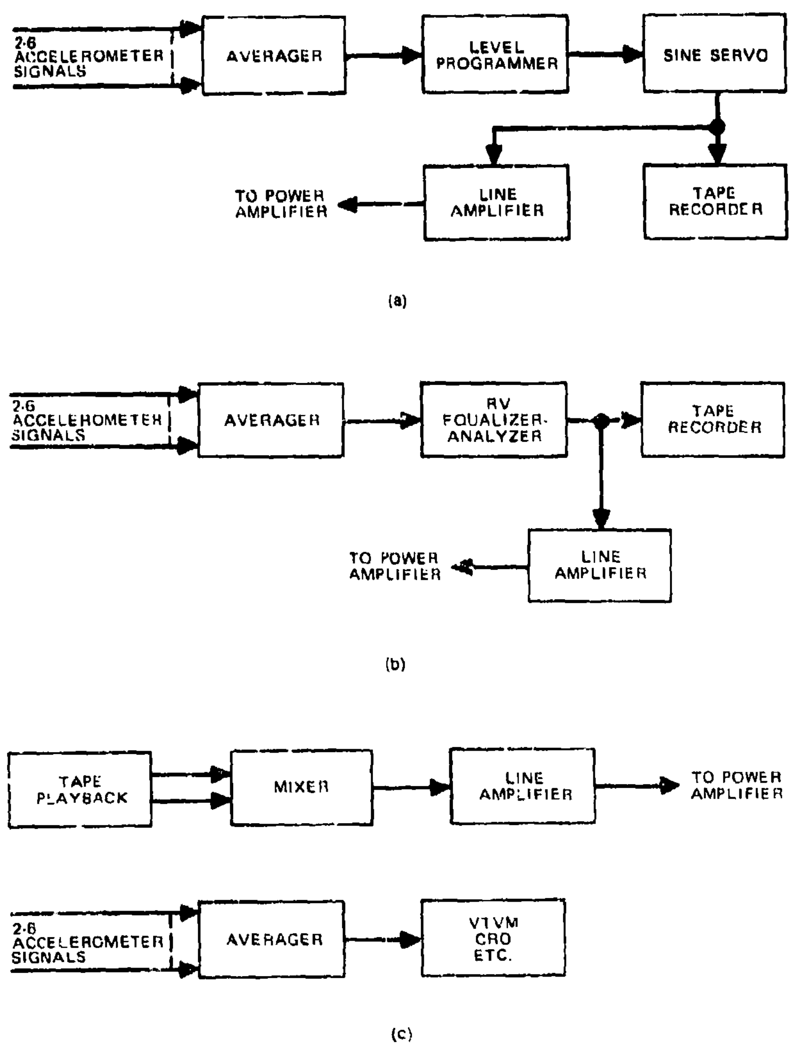

Fig. 5-12. Sinusoidal plus random test. Program tape preparation with random and sinusoidal averaging and sinusoidal level programming. (a) prerecording programmed and averaged sine excitation, (b) prerecording averaged and equalized random excitation, and (c) test performance.

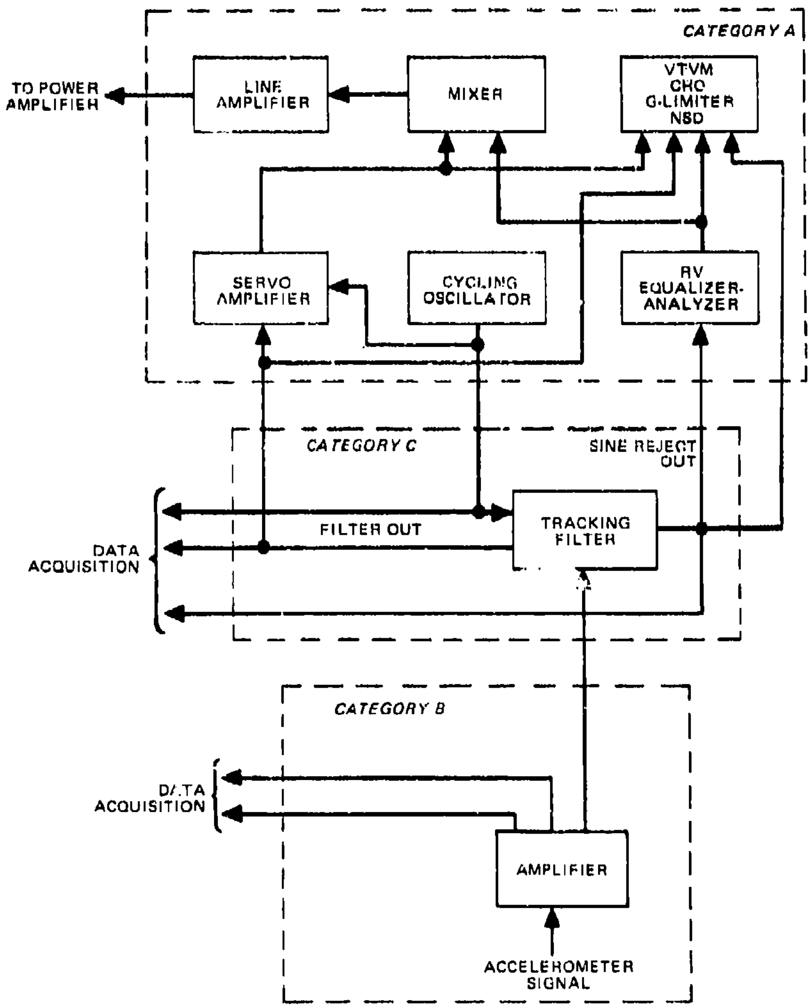

Fig. 5-13. Functional diagram of sinusoidal plus random test, tracking filter method.

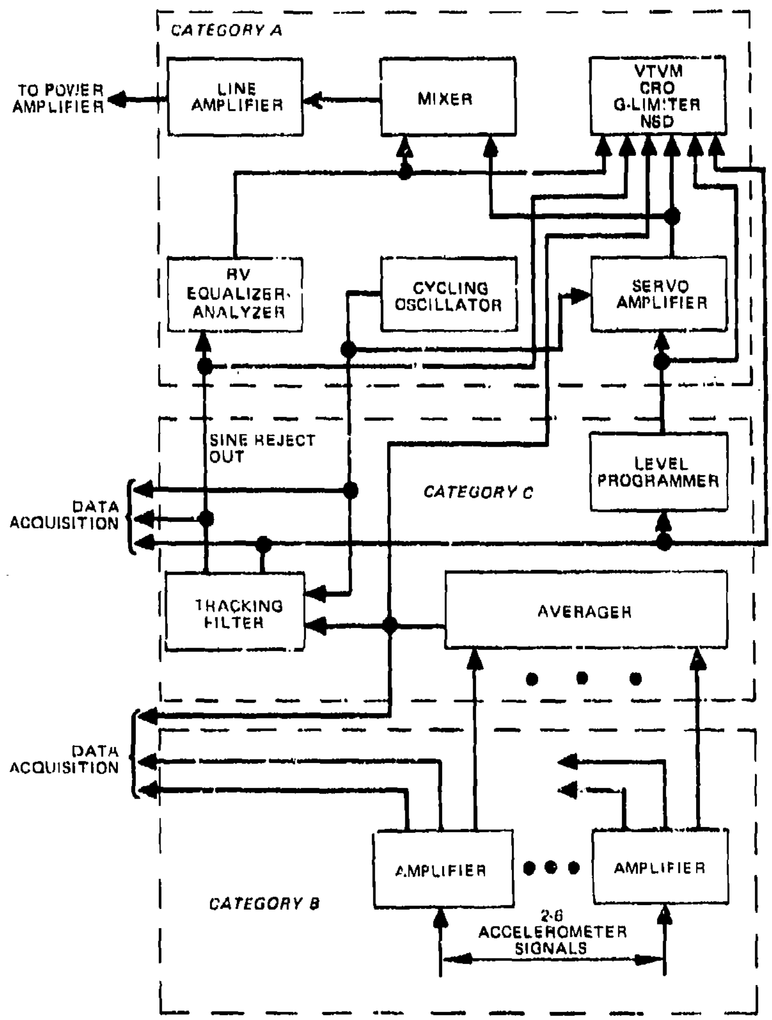

Fig. 5-14. Functional diagram of sinusoidal plus random test, tracking filter method with averaging and sinusoidal level programming.

laboratories were based on the use of audio equalizing networks and hastily devised peak and notch filters. Control methods ranged from simple broadband monitoring using a true rms voltmeter to more sophisticated use of various types of wave analyzers to permit at least some degree of control of the shape of the test spectrum. The usual excitation source was an electronic random noise generator, although some early tests were based on the use of magnetic tapes on which were recorded actual flight test data.

Commercial development of equipment customized to the task of random vibration testing led to steady improvement of the art. First to appear were multiply segmented, adjustable spectrum-shaping networks, and peak and notch filters with a fairly wide adjustment range for both Q and amplitude. The user was still left to his own devices for test control. Next to appear were the manual "equalizers" which consisted of matched fixed-frequency combination filter arrays: one, which had individual gain controls in each channel, was used for shaping the test spectrum; the other was used for determining the test level in the passband corresponding to each equalizing channel. These devices represented an enormous improvement over earlier methods but equalizing was still time consuming and usually left the customer with the unhappy conviction that test item malfunctions would really not have occurred had it not been for its pretest exposure.

The development of the "automatic equalizer," in which each channel is servo-controlled based upon feedback from the corresponding analyzer channel, has made the performance of random vibration tests relatively simple. Figures 5-15 and 5-16, respectively, diagram tests with single accelerometer control and those where control is based upon the averaged output of multiple accelerometers. For the latter case, the factors discussed on pp. 118-131 must be considered. The equalizing system must compensate for the electromechanical response characteristics of, and interactions between, the elements of the vibration system and the test mass. Since the required compensation at certain frequencies may often exceed the nominal 40-dB dynamic range of an equalizer in the automatic mode, there is usually provided in each channel an optional manual mode which adds about 10 dB to its dynamic range. The automatic equalizer also has a closed-loop operating mode which permits adjustment of the operating point of the servo in each channel prior to excitation of the vibration system. As a further precaution against unnecessary pretest exposure of the test item to vibration, the transition from closed-loop mode to test mode is made about 20 dB down from the test level, following which the test level may be increased as rapidly as the servo time constants allow or as slowly as desired. It is common practice to increase the level to −10 dB and then pause long enough to readjust servo operating points before proceeding to the full test level (where some final servo adjustments are often required).

A recent development for broadband random testing [97-99] involves the use of a digital system for excitation and spectrum shaping. The excitation signal is pseudorandom and is derived from a binary random noise generator; analyses

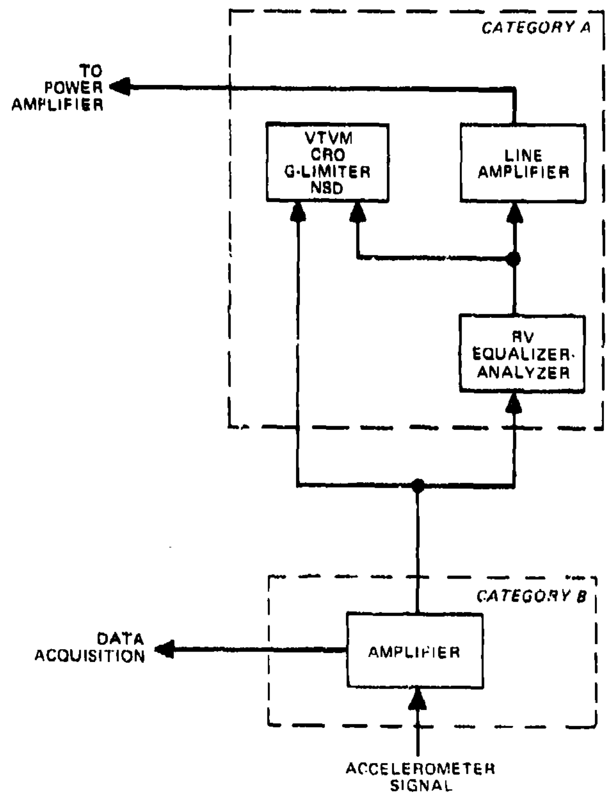

Fig. 5-15. Functional diagram for broadband random test with single accelerometer control.

of the control signal and spectrum shaping are effected through use of a special Fourier processor in conjunction with a small general purpose computer.

In estimating the required capacity of the vibration system it is usually necessary to calculate the rms g's, since test levels are ordinarily specified in terms of acceleration spectral density (g^2/Hz). For the uniform spectrum, the calculation is simple; i.e., it is the square root of the product of spectral density and test frequency bandwidth. However, for the shaped spectrum, the process is a bit more complicated if precision is attempted [100-102]. A useful approximation can be achieved by breaking up the sloped portions of the spectrum into sufficiently narrow frequency bands. It is derived by estimating the average spectral density (PSD) within each band, taking the products of bandwidths and corresponding PSD's, summing these with the products in the zero slope por-

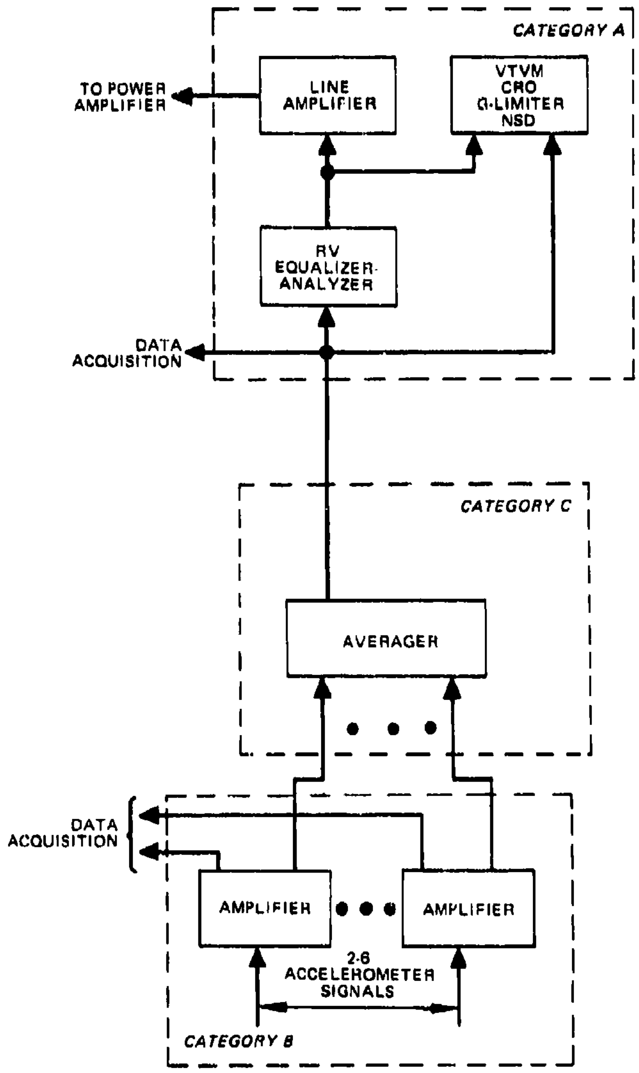

Fig. 5-16. Functional diagram for broadband random test with averaging.

tions of the spectrum, and taking the square root of the total. Appendix B contains formulas for precise calculations.

Swept or Stepped Narrowband Random Tests

The reader is referred back to Section 3.5 for the rationale for these tests. Either the swept or stepped versions are usually combined with broadband random base excitation but its presence or absence does not affect test implementation significantly. If the test is based upon equating broadband to narrowband cumulative fatigue damage, the base excitation is not required.

Swept Narrowband Random Tests. A typical test method is diagrammed in Fig. 5-17. The time required for sweeping the filter across its frequency range is an inverse function of filter bandwidth; as the bandwidth is decreased, both filter response time and detection time increase. The filter bandwidth and sweep rate are constrained also by test item response characteristics. The bandwidth must be great enough and the sweep rate low enough to permit adequate excitation of each test item resonance. These problems can be minimized in practice by switching progressively to wider bandwidth filters as the test frequency range is traversed upwards. An alternative approach is to use multiple tracking filters, with each one covering only part of the total frequency range. For information on other methods see Refs. 59 and 60.

If multiple control accelerometers are required, the methods discussed on pp. 118-131 may be used. However, the constraints on multiplexing are magnified due to filter sweeping.

Stepped Narrowband Random Tests. Many of the difficulties of the swept test are avoided by the use of the stepped narrowband random method, which is diagrammed in Fig. 5-18. Three tracking filters with dual-filter band-switching are used to cover the frequency range of 20 to 2000 Hz. Each of the six filters covers a limited frequency range, and the bandwidth of each is approximately 10 percent of its upper range. The excitation source is a program tape upon which are prerecorded the outputs and tuning signals of the tracking filters (with noise source inputs) as they are stepped and band-switched per schedule. During test performance, the tracking filters are connected in the accelerometer signal feedback path to servoamplifiers which operate on the corresponding prerecorded narrowband sources. Synchronized stepping and bandswitching is effected by using the prerecorded tuning signals to control the tracking filters.

Sweep rate is eliminated as a factor and, because the dwell time for each step is made very much greater than filter response time, the latter becomes negligible. The filter bandwidths chosen to cover each part of the test frequency range allow for adequate excitation of all test item resonances. See Ref. 61 for further information.

The averaging techniques described on pp. 118-131 may be used here also for multiple accelerometer control. For the multiplexing method, additional constraints due to filter traverse are minimized by the step dwell time.

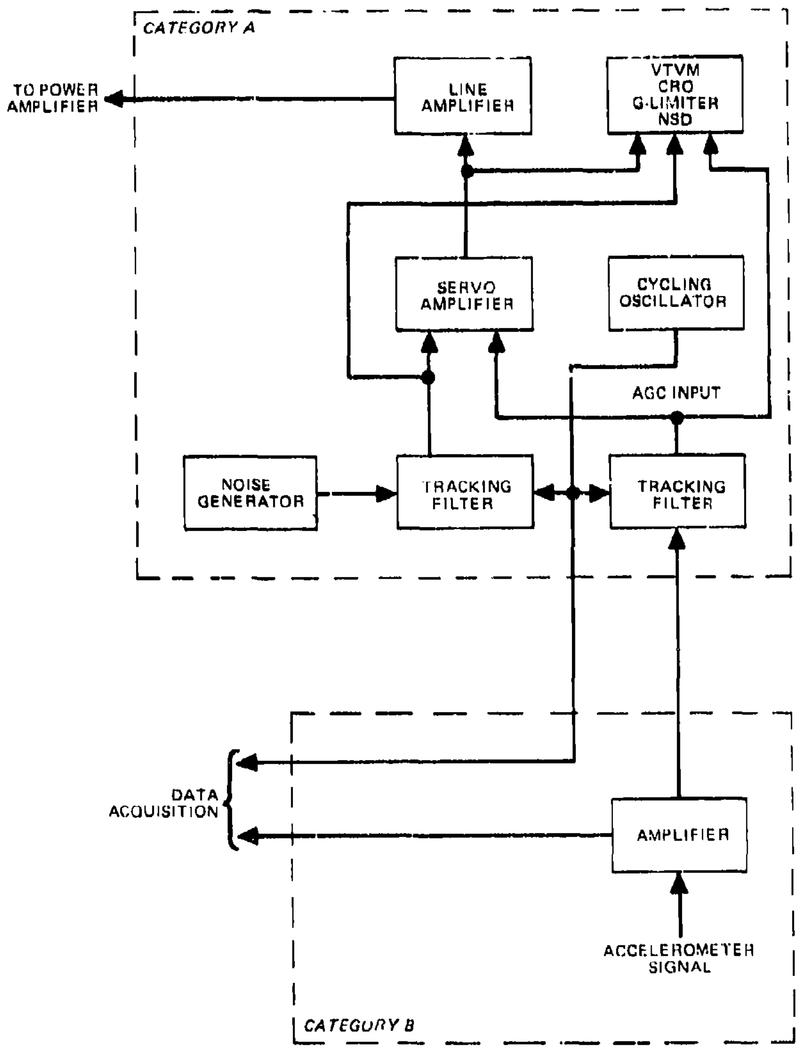

Fig. 5-17. Functional diagram for a swept narrowband random test.

5.5 Complex Waveform Tests

In these tests the form of excitation is not a simple sinusoidal function, but its time history is repetitive, or nearly so. Examples of such test applications are described in the following paragraphs.

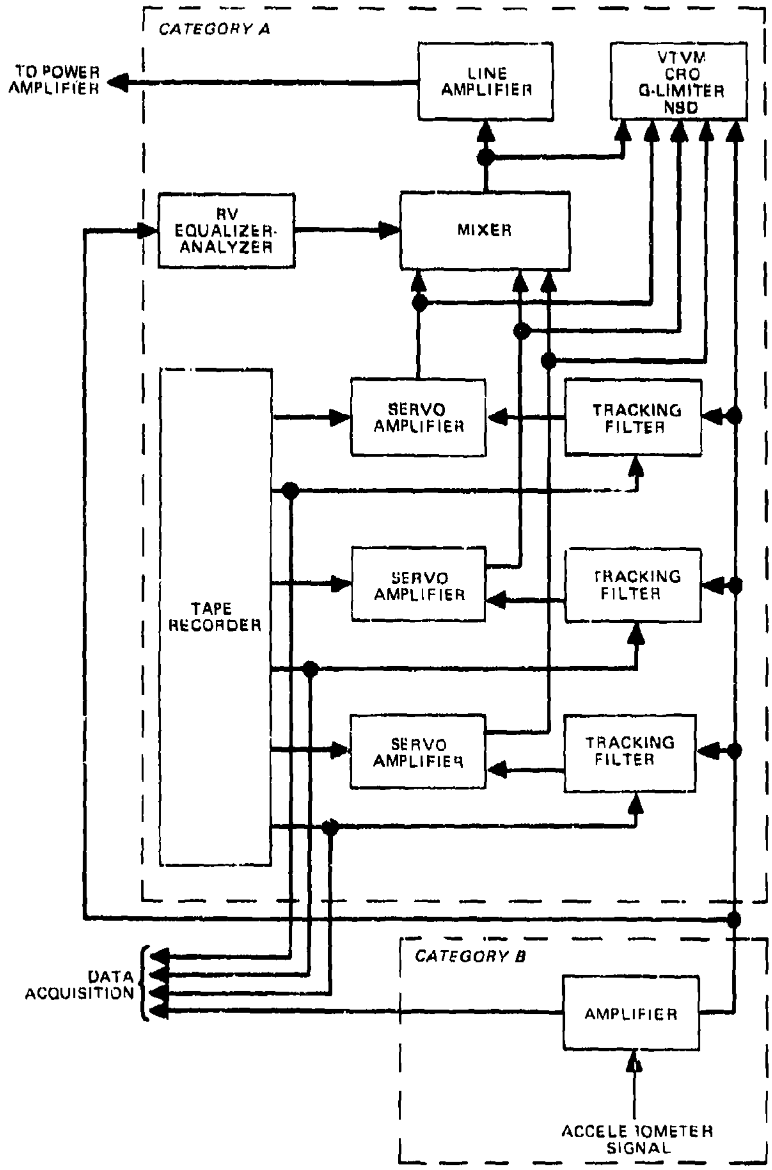

Fig. 5-18. Functional diagram for a stepped narrowband random test with broadband base excitation.

Reaction-Impulse Tests

Reaction-impulse tests are performed by using a hybrid reaction type of shaker which employs articulated eccentric weights (popularly called *clangers*) to superpose impulse loading on the reaction fundamental. The end result is a conglomerate of line spectra spread over a fairly wide frequency range. The shaker is designed to provide an inexpensive means of simulating partially the effects of random vibration and is used primarily for proof-of-workmanship tests [20, 21].

Test control is by indirection. The machine is mechanically adjusted to give the desired output for a specific semi-inert load. For later test use, the load is ballasted, if necessary, to the total mass for which the shaker was adjusted, and the test is run (usually uninstrumented) for the desired time. The equipment must be readjusted periodically to maintain the required output.

Pulsed Excitation Tests

Pulsed excitation is ideally suited to laboratory simulation of the effects of vibration induced by rapid-fire guns, as has been noted on pp. 99-102. A pulse train generator is used as the excitation source and its output is fed through a standard random vibration equalizer (Fig. 5-19). The latter is operated in the manual mode and is used to adjust the fundamental and harmonic amplitudes resulting from the pulse train. Equalizing is performed with random noise with the target spectral shape being the inverse of the pulse-train line spectrum rolloff as modified by the relative harmonic amplitudes of the desired test spectrum. If the equalizer is of the type with dual noise generators feeding alternate filter channels, an external noise source must be used. This is necessary to provide an equal degree of correlation between signals in the crossover region between adjacent channels for equalizing and for test performance.

Since the gun-firing rate usually varies above and below nominal due to fluctuations in hydraulic pressure and temperature, the ideal approach would be to sweep the pulse repetition frequency (prf) of the pulse train smoothly across the expected range of gunfire frequency. However, though sweeping can be achieved without too great a complication of test performance, the resulting task of certifying test levels (i.e., analyzing the control data) would be complicated enormously. A more practical approach is to step the pulse train prf across the range of firing-rate uncertainty N times, dwelling at each prf for 1/N of the total test time per axis. The number of steps is made large enough so that excitation frequencies will be no more than 1 to 1.5 dB down on the response curve for any test item resonance with a Q of 20 or less within the frequency range of interest, thus minimizing the chance of omitting damaging prf's.

In the case of airborne equipment, the gunfire-induced vibration is usually accompanied by random excitation. In performing the test, it is relatively easy to provide the latter. Test control is based upon monitoring the control signal

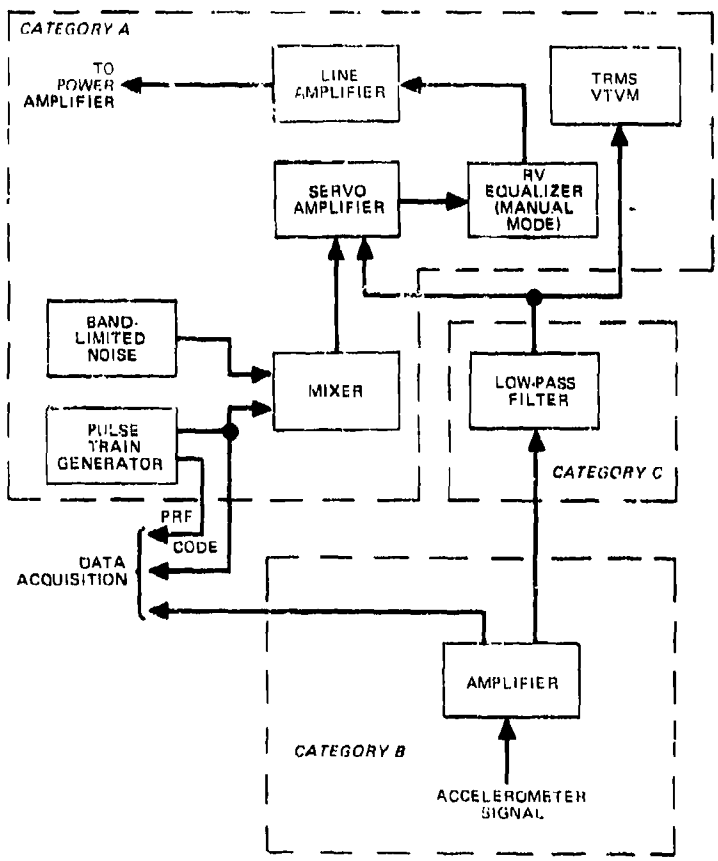

Fig. 5-19. Functional diagram for pulsed excitation test with broadband random base excitation.

and adjusting the gain to maintain the desired rms level. If control is based on multiple accelerometers, the multiplexing method can be used for averaging. If the control accelerometers are not physically oriented so that their signal polarities are identical, the precautions of Section 4.3 must be observed.

5.6 Response-Limited Tests

For some test structures it may be desirable to modify the input in order to limit the maximum response at one or more points on the structure [1]. The basic test methods described in previous sections can be modified to achieve this end.

Sinusoidal

The simplest technique for limiting the response for sinusoidal tests is to use signal selection and to include in the selector inputs the signals from the accelerometers mounted at the points at which limiting is desired. It is also possible to combine control accelerometer averaging with response limiting. The method is diagrammed in Fig. 5-20, where it will be noted that the output of the multiplexer is one of the inputs to the signal selector. However, in the selector channel used for this purpose, the detector averaging time must be at least as great as the time required for one full cycle of the multiplexer. If averaging is used, the servo time constant also should be no less than the cycle time, or instability is likely to occur whenever the averager channel is selected; the maximum sweep rate is limited by the minimum permissible servo time constant. If averaging is not used, filtering may be employed as in Fig. 5-8. It is recommended that averaging and filtering be avoided.

Random or Pulsed Excitation

In general, response limiting for random or pulsed excitation tests can be achieved only by iterative excitation of the test item. After each iteration, recorded accelerometer outputs are analyzed and responses compared to the input(s) to determine what input modifications are required. Iterations are started about 10 dB down from the test level to minimize pretest stressing of the test object. Averaging can be applied but the constraints of Section 4.3 on pp. 118-131 must be observed.

The obvious disadvantage to this technique is the waiting time between iterations while analysis, comparisons, and input adjustment calculations are taking place. If a high-speed digital analysis system is available, the waiting time can be reduced to acceptable limits; for further details refer to page 196.

5.7 Multiple Shaker Tests

The earliest application of multiple-point excitation is exemplified by the use of small reaction vibrators for structural testing of complete aircraft prior to World War II. Multiple excitation over a relatively wide frequency range is a technique that has been developed in recent years with the advent of high-performance aircraft and large space vehicles. Since adequate coverage of the topic is beyond the scope of this monograph, only a few general observations are presented below; however, Refs. 5 through 16 contain considerable information regarding techniques and problems associated with multiple excitation.

The basic difficulties are facility costs and the complex problem of test level control. For sinusoidal testing, for example, both amplitude and relative phase must be controlled at the input points. For random tests, the control problem is reduced somewhat if separate excitation sources and equalizers are

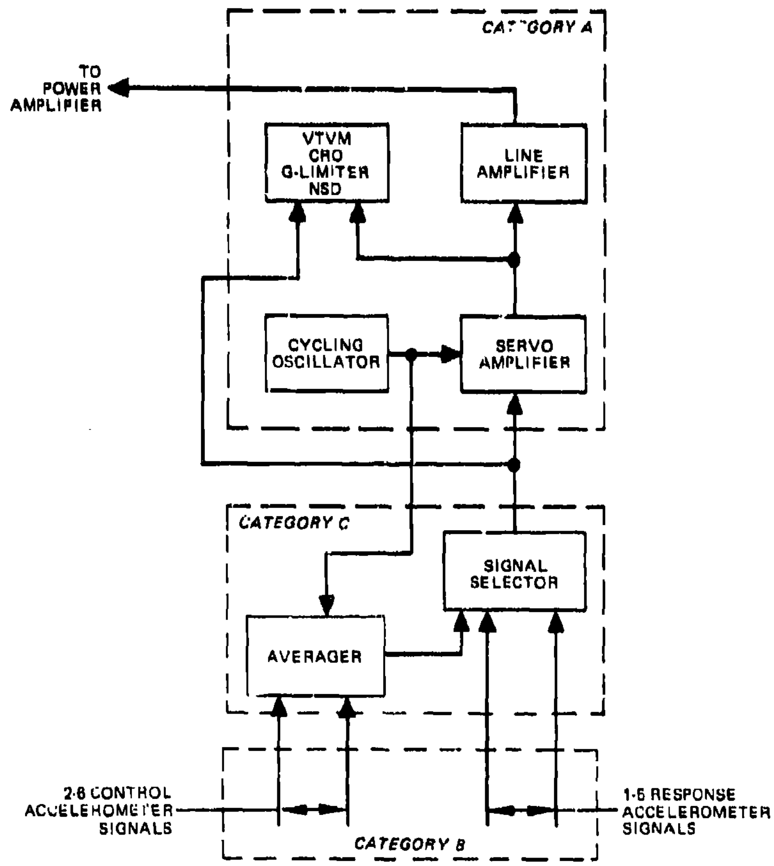

Fig. 5-20. Functional diagram for sinusoidal test, response-limited, with signal selection between response and averaged control accelerometers.

used, but the equipment cost is very large. In either case, any significant cross-coupling between excitation points can complicate enormously the problem of maintaining the desired test level at each point.

CHAPTER 6
ACQUISITION AND PROCESSING OF TEST DATA

It may be generalized that the sole end product of any test is the data resulting from the test. The measurement of vibration test results against success criteria must be based upon data falling into two basic categories: (1) the vibration levels experienced by the test item at its various points of interest and relations between these, and (2) the performance of the test item. In the latter case, criteria may range from simple physical survival to complex functional requirements. The material in this chapter is restricted to the recording, processing, and presentation of data in the first category. The basic assumption is made that the end aim with respect to these data is to convert them to the form or forms most suitable to the task of evaluating how well test objectives were met.

Workers directly involved in vibration test performance tend to regard the handling of vibration test data as a procedure consisting of acquisition and analysis (the latter term being applied to the entire process of converting the raw data to its final form of presentation). However, as a concession to the dynamicist, who reserves the term *analysis* for the cerebral processes applied to the end result of data processing or data reduction by the test laboratory, the basic data functions covered here are defined as acquisition, processing, and presentation.

A very complete discussion of the acquisition, processing, and presentation of vibration data would go far beyond the needs of this monograph, and the reader should refer to Refs. 25, 32, 33, 56, and 103, for example, for more detailed discussions. The discussion herein will be confined to that necessary for an understanding of the requirements for acquiring, processing, and presenting data from laboratory vibration tests. These requirements are much less stringent than those for the acquisition and processing of data from field measurements for two basic reasons: First, the vibration levels are generally either known or can be accurately estimated before test. Second, the statistical characteristics of the data are known, i.e., sinusoidal, complex or random, Gaussian, stationary, etc., so that simplified procedures for editing and processing can be employed.

6.1 Data Acquisition

In the sense used here, acquisition is a combination of signal conditioning and recording functions. Conditioning is defined as the modification applied to analog signals to convert them to a form that can be recorded and translated correctly into engineering units for the parameters represented. Recording may be done on magnetic or oscillographic media or it can be as simple as meter reading and hand logging of the readings by an operator. It is obvious that the utility of the recorded data can be impaired seriously by errors in conditioning.

The value of the recorded data depends also upon the ability to correlate it with specified test levels and test item performance phenomena. Aside from the obvious requirement to annotate records with information such as vibration axis, transducer location and orientation, sensitivity, scale factor, etc., means should be provided for relating the data to time or frequency.

Signal Conditioning Factors

Two assumptions are made with respect to the control and monitoring instrumentation: (1) that the transducers have been calibrated properly and (2) that the required corresponding sensitivity settings have been made correctly for each transducer amplifier. The resulting signals (for accelerometers) will usually have a sensitivity of 10 mV/g. If oscillographic recording is required, power amplification is necessary to provide sufficient current to drive the galvanometers and to match their impedance. If magnetic tape recording is required, voltage amplification is usually necessary to obtain a satisfactory signal-to-noise ratio for later data processing.

Much of the signal conditioning equipment developed in recent years includes, in addition to a fixed 10 mV/g (often called the *servo*) output and a meter indicating the g level, the current and voltage amplification channels required for recording applications. Each of the latter two has a gain control permitting adjustment of the analog sensitivity of the recordings. However, on most such instruments there is a meter range switch which also affects the amplitudes of the recorder outputs.

Oscillographic Recording

In the early years of vibration testing, direct readout recorders could be used only for low-frequency data (to about 200 Hz). To capture higher frequency data it was necessary to use recorders writing on photographic paper, which required later darkroom development and drying before the records could be read. Considerable skill and a measure of luck were prerequisites to obtaining complete and readable data; the delay between test performance and analysis was frustrating and often costly.

With the advent, in the early 1950's, of the direct readout oscillograph with considerable latitude in light beam intensity, realtime recording of vibration test data became a fairly routine operation. Unfortunately, the relative simplicity of the technique makes it as easy to misapply as to use correctly. A wide range of galvanometer types is available with different drive current, impedance, and nominal frequency response characteristics. The latter two are interdependent in the sense that the nominal frequency response is obtained only if the drive amplifier matches the galvanometer impedance. It is obvious then that the galvanometer and drive amplifier ideally should be matched to the job; i.e., they should be selected on the basis of data frequency response requirements. In

practice, the current amplifier channel of the usual signal conditioner has a fixed output impedance which may or may not match the requirement for the galvanometer used. Therefore an impedance matching device should be inserted between the current amplifier and galvanometer. The device may consist of a simple passive impedance transforming network or an isolation amplifier designed for the purpose; the latter is preferable since it assures sufficient galvanometer drive without signal limiting due to saturation of the current amplifier.

As has been noted earlier, the value of the data is impaired if the user cannot relate the records to time and frequency. This information can be approximated by operator annotation of the record with chart paper speed initially, and sweep frequency (for sinusoidal test) often enough to allow later interpolation. The better method is to record frequency coding on a channel of the oscillograph. Commercial equipment is available for this purpose; essentially it is a counter with serial output of four ten-step dc staircase voltages, with each staircase representing a decimal digit from 0 to 9. Thus, frequencies from 0 to 9999 Hz can be coded and recorded. If an IRIG time standard or generator is available with an IRIG C ("slow code") output, coded time can also be recorded automatically on the oscillograph. Many oscillographs make internal provision for placing 1-sec or 0.1-sec timing lines on the record. Fig. 6-1 shows a sample record with both frequency and time codes.

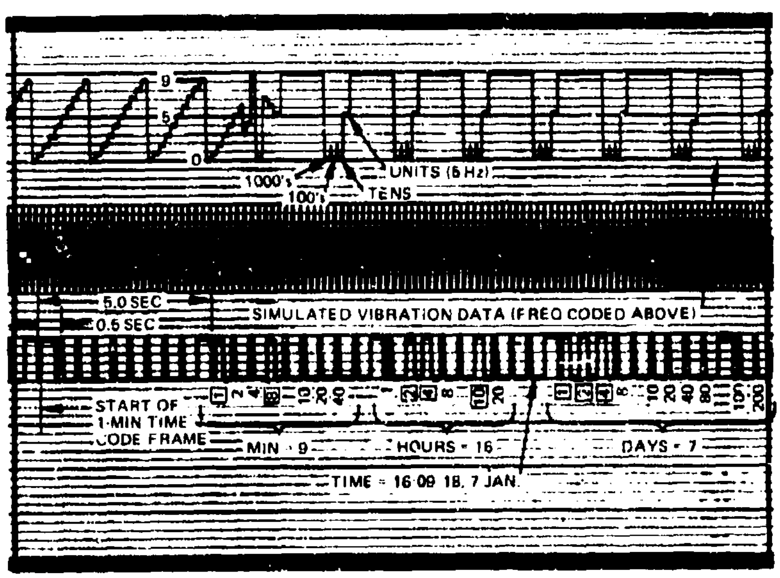

Fig. 6.1. Sample oscillographic record with frequency and time coding.

Magnetic Tape Recording

There are three basic techniques used for tape-recording vibration data: direct, frequency modulation (FM) and frequency division multiplexing (an extension of FM). With the first technique, the analog signal is recorded directly on tape with format unchanged. For a given tape speed the high-frequency response is considerably greater than for the other two; however, low-frequency is degraded below 50 to 100 Hz and flatness of response depends upon the precision with which record and reproduce equalizing is applied. In FM recording, the amount of carrier frequency deviation is proportional to signal amplitude and the rate of deviation is determined by the frequency content of the signal. Frequency response is reasonably flat from zero (dc) to an upper frequency which depends on tape speed, carrier frequency, and the signal-to-noise ratio (S/N) that can be tolerated. Table 6-1 illustrates the relationship of these parameters for various IRIG standard recording systems.

In frequency division multiplex recording, each signal to be recorded on a tape track is fed to a separate voltage-controlled oscillator (VCO). Each VCO has a different zero-signal frequency which is deviated (frequency modulated) by its input signal. The modulated carriers are then mixed and direct recorded on tape. Special demultiplexing equipment is required to separate and recover the individual data signals for processing. The number of channels that can be recorded on a track is a complex function of recorder characteristics, tape speed, data bandwidth per channel, permissible noise level, etc. The technique was derived from a method originally developed for telemetering flight test data to ground stations; consequently, much of the information in the literature (e.g., Ref. 104) cannot be applied easily to the task of recording laboratory test data. However, the technique has been applied to the airborne recording of captive flight environmental data. Two such cases noted below for captive missile flight give a partial indication of the amount of data that can be recorded on a single tape track.

1. Seven channels of data with nominal frequency responses of 500 Hz in four channels and 2000 Hz in three channels (Ref. 105).

2. Five channels of data with nominal frequency responses of 300 Hz in two channels, 500 Hz in one channel, and 2000 Hz in two channels (Ref. 106).

Given an extended bandwidth recorder and reasonable care in applying the technique, it is possible to place 8 to 10 data channels of 2000-Hz bandwidth on one tape track. The basic limiting factor is noise in the higher frequency carrier channel outputs. For constant data bandwidth, the maximum carrier deviation is the same for each channel; thus, with increasing carrier frequency, the percentage of deviation and the demultiplexing discriminator full-scale output become progressively smaller, so that relative noise in the discriminator outputs becomes progressively larger.

The maximum signal amplitudes that can be handled in direct recording vary widely, depending on the individual recorder and electronics. In general, it may

Table 6-1. FM Record/Reproduce Frequency Response vs Tape Speed (IRIG Standard)

Tape Speed (ips)	Standard Bandwidth (BW) (IRIG Low Band)			Extended Bandwidth (BW) (IRIG Intermediate Band)		
	Carrier Freq. (kHz)	1.0 dB BW (kHz)	RMS S/N (dB)	Carrier Freq. (kHz)	1.0 dB BW (kHz)	RMS S/N (dB)
120	108	0-20	51	216	0-40	49
60	54	0-10	51	108	0-20	49
30	27	0-5	51	54	0-10	49
15	13.5	0-2.5	51	27	0-5	49
7.5	6.75	0-1.25	51	13.5	0-2.5	49
3.75	3.375	0-0.625	49	6.75	0-1.25	47
	Double Extended Bandwidth (IRIG Wideband Group 1)			Extra Bandwidth (BW) (IRIG Wideband Group 2)		
Tape Speed (ips)	Carrier Freq. (kHz)	1.0 dB BW (kHz)	RMS S/N (dB)	Carrier Freq. (kHz)	3.0 dB BW (kHz)	RMS S/N (dB)
120	432	0-80	49	900	0-400	32
60	216	0-40	49	450	0-200	32
30	108	0-20	48	225	0-100	30
15	54	0-10	47	112.5	0-50	28
7.5	27	0-5	47	56.25	0-25	26
3.75	13.5	0-2.5	45	28.125	0-12.5	26

be assumed that much larger signals are possible in the direct mode as compared to the FM mode. For the FM recorder, maximum signal amplitudes are limited by the modulation technique rather than by the characteristics of the head configuration and recording media. Normally, the maximum frequency deviation of ±40 percent is equivalent to a maximum signal excursion of about ±1.4 V. If these limits are exceeded, discriminator operation becomes nonlinear and unpredictable errors will be introduced into the processed data.

Regardless of the recording mode used, the acquisition process should include the following basic steps:

1. In a tape log sheet should be entered information sufficiently detailed so that the data on each track can be identified unambiguously for later processing.

Certain types of general information (e.g., test item, test axis, date, time of day, etc.,) are often voice-annotated on one of the tape tracks. There are advantages to this which must be weighed against the loss of vibration data recording capacity.

2. Shortly before the test is started, a reference, or calibration, signal should be recorded on each data channel. These signals must be related to the system analog sensitivity of each instrumentation channel. As a typical example, a sinusoidal signal of 1.0 V rms might represent 10.0 vector g's.

3. After start of test, if range switching is required because of unexpectedly high or low signals in one or more data channels, the direction, amount, and time of change must be entered in the log unless some scheme for automatic range coding is being used. The latter is available as an option with some signal conditioning equipment. It should be noted, however, that part of the dynamic range is lost thereby.

The importance of these steps cannot be overemphasized since the validity of all subsequent data processing will depend upon the accuracy of the reference signals and pertinent notations on the tape log.

To minimize test support costs, it is common practice to reuse magnetic tape after data processing is completed. In theory the tape recorder either provides for erasure prior to recording (direct) or erases as it records (FM); however, the process often leaves a vestigial imprint of the prior record which shows up as unwanted noise in the new record. For this reason it is recommended that bulk degaussing be applied to tapes before reuse; the required equipment is commercially available and relatively inexpensive.

The fidelity with which the data are recorded and later translated back into meaningful forms during processing also depends on (1) the care used in alignment of record and reproduce electronics, (2) proper alignment of the tape transport and heads, (3) cleanliness of record and reproduce heads and (4) the quality of the tape. The first factor applies to each use of the recorder and the next two are preventive maintenance factors. The last factor places a limit on tape reuse; further definition is impossible since it depends on original tape quality, tape handling and storage, the specific recorder(s) on which it is used, and the quality of preventive maintenance.

The separation and identification of test phenomena during data processing are simplified greatly if a time code data channel has been recorded; the IRIG B code is ideally suited for the purpose. It should be recorded directly, if possible, to allow use of tape search and control equipment. There are other supplementary data which must be recorded on tape to reduce costs or increase the scope of data processing. In the following paragraphs, these additional requirements are described for particular types of tests.

1. Sinusoidal tests. As a general rule, the sweep oscillator output should always be recorded on one data channel of each tape. It will be needed for measuring relative phase and for tuning the tracking filter(s) for transmissibility

measurement. If a filtered sweep is used, recording the output of the tracking filter will reduce the complexity and cost of transmissibility measurements because only one tracking filter is then needed for data processing. If signal selection is used, a code identifying the controlling channel should be recorded if available.

2. Swept or stepped narrowband random tests. There will be one or more tuning or control signals which must be recorded for later use in data processing.

3. Pulsed excitation tests. As will be seen in Section 6.3 (page 186), processing the data from these tests is greatly simplified if two added information channels are recorded: the pulse train source and a code identifying the prf.

6.2 Data Preprocessing and Editing

Data preprocessing and editing are those procedures used to modify and select, respectively, the analog data signals recorded during test prior to the data processing procedures which transform the analog signals in some other form.

Preprocessing procedures are applicable primarily to tape-recorded data but may be applied (sometimes inadvertently) to oscillographic records. For example, if data above some frequency, say 1000 Hz, are not required for test evaluation, low-pass filters might be inserted in the inputs to the oscillograph. The resulting records are easier to read with the unneeded frequency content removed. A similar effect can be achieved by the use of galvanometers that have limited response characteristics, but frequency rolloff will be much more gradual. If the signal amplitude at the test excitation frequency is to be directly readable (without Fourier analysis), narrowband filtering must be applied before oscillographic recording. It is likely to be more economical, however, to tape record the test data, and filter and generate oscillograms after the test is complete. Similarly, data may be preprocessed by filtering before being tape recorded. This procedure should be avoided except under extraordinary circumstances since it can be effected so easily during processing of the taped data.

Editing is defined as the procedures used to locate wanted data in the records; to identify corresponding parameters such as time or frequency bounds, sensitivities or scale factors, data sources, etc.; and to provide "quick-look" data presentation. The basic editing tools for taped data are the oscilloscope and oscillograph used in conjunction with the tape log and supplemental recorded data. Preprocessing such as filtering may be applied also. It is often desirable, if it was not done during test, to record a time code before data editing. If data volume is large or extensive processing is required, it is good practice to dub working tapes from the originals to avoid data degradation as a result of tape wear. If analog processing is to be applied to short sequences of test data, the corresponding tape segments must be cut and spliced into loops permitting iterative playback. High-quality splicing is required to minimize the introduction of spurious signals as the splice passes the reproduce head.

6.3 Data Processing and Presentation

Data processing and presentation are most conveniently treated as one subject because they are often inseparable functions. Except for the first subsection, which deals with factors either applicable to data processing in general or related to multi-use processing equipment, the material is discussed separately for the several types of tests.

General Considerations of Accuracy

For the purposes of this discussion, *accuracy* is a term used loosely to encompass factors which the purist will insist on separating into categories such as precision, accuracy, resolution, etc.

The first thing to be noted here is that, since there are bound to be certain limitations on data quality inherent in the acquisition procedure, processing accuracy requirements should be reasonably related to these limitations. For example, it is senseless and unnecessarily costly to require 0.1-percent processing accuracy for 5-percent data.

Most laboratories are subject to quality assurance requirements for periodic calibration checks and certification of some of the instrumentation used for data processing. Use of such equipment when it is near or beyond its recalibration date is poor practice unless the validity of processing results can be demonstrated unquestionably.

The following paragraphs identify, by types of processing functions or equipment, general factors affecting accuracy.

1. Magnetic tape reproduction. Assuming that the precautions cited on pp. 174-176 have been duly observed, preservation of data quality in processing first requires careful alignment of reproduce electronics. Next, the analog sensitivity of the data must be determined from the reference signal and tape log. These steps must be performed with all equipment that will be used connected into the processing system to avoid the introduction of errors due to loading effects.

2. Tracking filter. This device is a common primary element in test data analog processing for phase and transmissibility measurements, wave analysis, power spectral density estimates, etc. For every such use, the following precautions should be observed:

(a) The filter bandwidth should be selected to match data and processing requirements. For example, if random test data are being processed, required sweep and detection times increase with decreased filter bandwidth to permit the processing system to respond to changes in the power spectral density of the data signal.

(b) Instructions for alignment should be followed completely and carefully. This innocent-appearing instrument is a very complex device which performs poorly if not properly adjusted.

(c) Final system sensitivity measurement (and adjustment, if necessary) should be made with all input, output, and monitoring equipment connected.

3. Miscellaneous. Other equipment commonly used, such as xy plotters and log converters, present potential problems. For these and other equipments, the arrangement for range changing or gain/attenuation setting is often such that interface impedances are changed also. For this reason, once the final processing system alignment and sensitivity are determined, no changes in range, gain, etc., should be made without rechecking system analog sensitivity. An additional factor applicable to most processing equipments is related to their dynamic range. In general, for any given application, there is an optimum operating region; if signal amplitudes are consistently low or high relative to optimum, errors may be introduced due to unwanted noise or signal limiting, respectively.

Sinusoidal Tests

Real-Time Processing. On-line oscillographic recording of unmodified data signals is a common form of treatment for data from either filtered or unfiltered swept sinusoidal tests. This is obviously little more than data acquisition, but the records, suitably annotated, are often the nearest the test data comes to being processed. The procedure may be adequate for many tests with the limited objective of determining if the test item can survive, in physical or functional terms, exposure to controlled vibration levels. However, if the test item fails or exhibits anomalous behavior, an explanation is usually sought in the recorded data. If the test control signal was unfiltered, analysis of test item behavior is an impossible task unless very fast chart paper speed was used; in the latter case, the physical record length may require adjournment to the nearest long corridor for visual and manual analysis! For all but the most routine unfiltered sweep tests, if the data cannot be tape recorded, it is recommended that the filtered (in addition to the unfiltered) control signal be recorded on the oscillograph. Fig. 6-2 diagrams a typical setup for doing so. The second output of the tracking filter shown is available as a standard option and is recommended also for its diagnostic value. It is commonly called the *sine reject* output and is a broadband signal with the sweep excitation frequency notched out.

Figure 6-2 also shows the alternative use of a dual-channel xy plotter for recording the processed control signal; this figure is an example of such processing. Two plotters can be substituted if a dual-channel device is not available. This is a particularly useful technique for the single-sweep test but may be used also for periodic sampling of the multisweep test. The x-axis log converter permits representation on standard log graph paper. The y-axis log converters serve a dual purpose: detection of the data signals and increasing the dynamic range of signal amplitude presentation. If linear presentation is required, detectors and plotter drive amplifiers must be substituted for the log converters.

Limited on-line transmissibility plotting may be performed also if the appropriate equipment is available. Figure 6-3 diagrams the preferred method

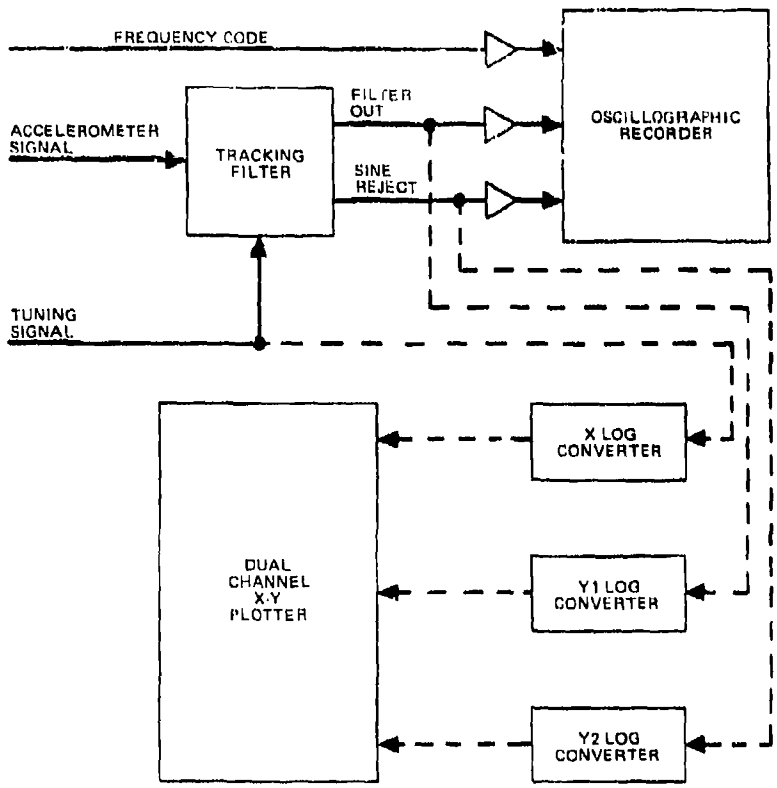

Fig. 6-2. On-line data processing, swept sinusoidal test (unfiltered).

which ratios amplitudes at the fundamental sweep excitation frequency only for a response signal and the control signal, respectively. For a filtered swept sinusoidal test, the tracking filter shown in the control signal path is the one which is used for test performance. A ratio of the signals is obtained by simply reversing the polarity of one of the log converters.

Other transmissibility techniques, which do not discriminate against the harmonic-distortion content of the signals, can be used. Specialized plotting devices are available, or a multiple signal oscillographic recording method [107] can be applied. However, the meaning of the ratio of two signals which have not been filtered is, to say the least, unclear.

Taped Data Processing. The simple processing described in the previous section can, of course, be applied to all taped data. The resulting records can be made much easier to use in the analysis of test performance, since the editing

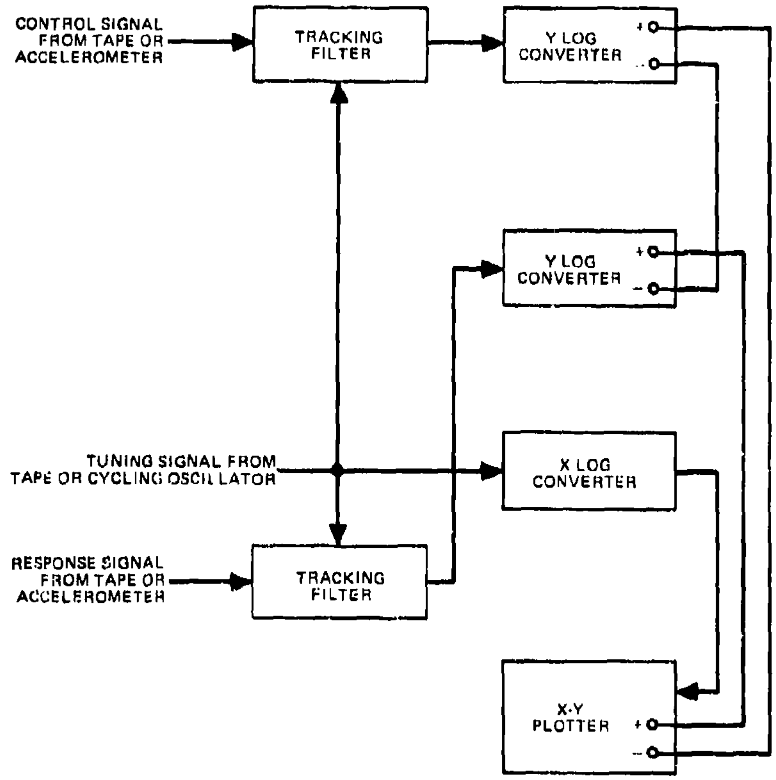

Fig. 6-3. Transmissibility plotting, tracking filter method.

techniques on page 177 may be applied also. For the filtered swept sinusoidal test, if the filtered signal was not recorded, transmissibility plotting requires the setup diagrammed in Fig. 6-3. If the filtered control signal was recorded during test, one tracking filter can be eliminated and the reproduced control signal fed directly to the corresponding log converter.

Sinusoidal Plus Random Tests

It is improbable that the tape programming method (see pp. 154-156) will be used if a tracking filter is available. Therefore, processing options for such test data, whether tape recorded or not, are likely to be restricted to the simple expedient of outputting the broadband signals on a plotter or an oscillograph as a function of time. It is conceivable, however, that tape-recorded data might be processed at another time or place where a tracking filter is available; in that case

the technique described in the next paragraph may be appli l if a constant-amplitude sweep frequency signal has been recorded for tracking filter tuning. The recorded sweep excitation signal usually will be unsatisfactory for the purpose because its amplitude variations at some frequencies will exceed the allowable range for reliable tracking.

On-line data processing for the test using the tracking filter method (page 156) is diagrammed in Fig. 6-4. Two basic graphic presentations are generated: a plot of power spectral density averaged over the equalizing filter bandwidths, and a plot of sinusoidal amplitude vs frequency. Only the control signal can be processed on-line unless the test laboratory is blessed with both multiple tracking filters and xy plotters and spare equalizing systems. The second output (sine reject), which is shown connected to the xy plotter via the mean-square detector, is an option which may have diagnostic value if test control anomalies occur. The tracking filter shown is, of course, the same one used for test control.

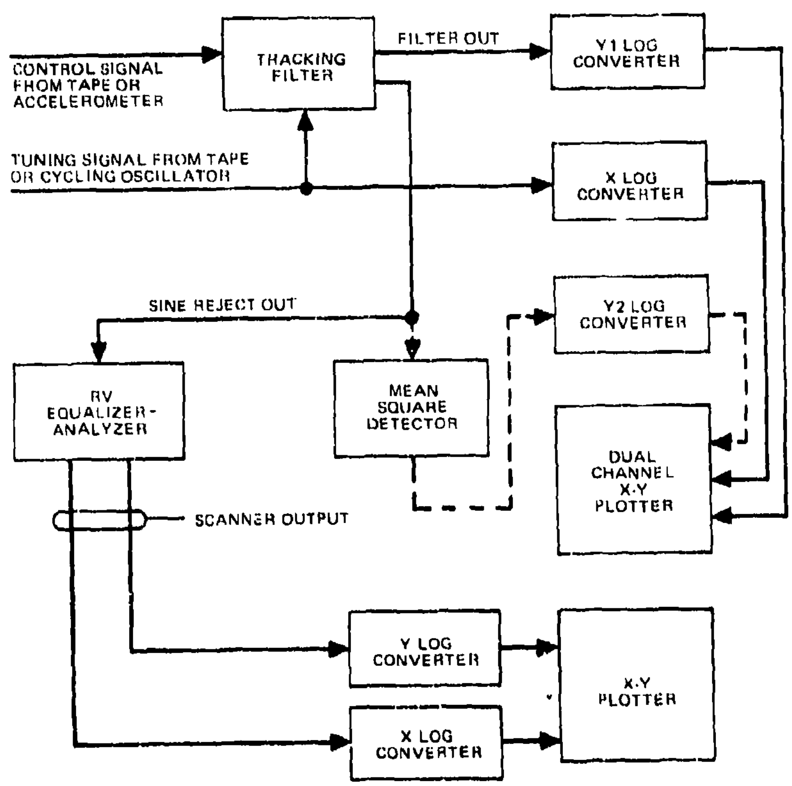

Fig. 6-4. Sinusoidal plus random test data processing.

For processing tape-recorded data, the only difference is that tracking filter inputs come from the tape recorder rather than from test control equipment.

Broadband Random Tests

Processing of data, either on line or from recordings, obtained during random vibration tests entails what is generally known as spectral analysis. The type of spectral analysis performed will be very dependent on the equipment available and the intended use of the data subsequent to processing. Chapter 2 contains discussions of some of the parameters which must be considered in selecting the spectral analysis method, particularly the filter bandwidth to be employed. In this section and following sections regarding specialized random vibration tests, it will be seen that there are two categories of spectral analysis. The first is that carried on continuously within the vibration test equipment in order to control the test. It is only necessary to read out the spectral values at an appropriate time. The second is that carried out either during, or more often, after the conclusion of the test, using an available spectral analyzer which is not an integral part of the vibration test equipment. The spectral analyzers can be classified as constant-bandwidth (swept filter, constant-percentage bandwidth) comb filter, and special purpose. The following subsections briefly describe the use of these types (or categories) of spectral analysis for broadband random vibration tests.

Equalizer-Analyzer System. Most random vibration equalizer-analyzer systems present a continuous visual display of the spectral density in each equalizer channel. In addition a scanner or commutator which samples a voltage representing the spectral density in each channel is included in the system. Figure 6-5 illustrates the equipment necessary to obtain an xy plot of this spectrum. Of course, tape recordings of random vibration signals can always be played back through the analysis section to perform this type of analysis, providing the signal is attenuated to obtain the normal 10 mV/g sensitivity.

Constant-Bandwidth, Swept-Filter Analysis. This type of data reduction may be performed on line or by using taped records. It is usually accomplished by effectively sweeping a filter across the frequency range of interest, but is sometimes done by stepping the filter incrementally. There are various equipments such as tracking filters, wave analyzers, and other specialized instruments that may be used for the purpose. A typical processing setup is diagrammed in Fig. 6-6. Regardless of the equipment used, there are three basic factors to be considered.

1. In choosing the filter bandwidth, it must be remembered that both the allowable sweep and the maximum theoretical accuracy are limited thereby; sweep rate because of filter response and detection time constraints, and accuracy because there is an inherent statistical error which is a function of bandwidth and sampling time [33,108].

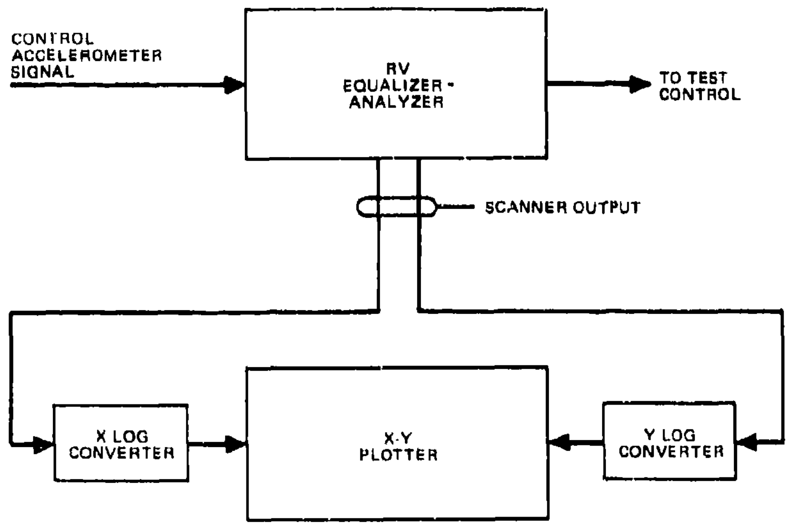

Fig. 6-5. Broadband random test data processing, on-line.

2. The effective bandwidth of the filter must be known in order to align the system properly unless an accurate reference noise source is available.

3. The filter sweep rate and detection time constant must be chosen carefully to minimize "grass" on the plot and still maintain the ability of the system to respond to variations with frequency of the spectral density [108].

If the equipment is available, digital detection and processing of the filter output can be performed. The procedure is particularly attractive if the system provides for digital incremental plotting, since a large data volume can be processed rapidly into its final presentation format. However, there are programming complications and uncertainty in frequency determination if the filter is swept; for these reasons, it is recommended that the filter be incrementally stepped for digital processing.

Constant-Percentage-Bandwidth, Comb Filter Analysis. When constant-percentage bandwidth analysis is desired, swept-filter processing is not recommended since economical technology, which maintains filter quality and also allows continuous bandwidth change, does not yet exist [108]. The obvious alternative is to use a combination arrangement of fixed-frequency filters overlapping so the response curves for adjacent filters intersect approximately at their −3 dB points. An early version of this technique [109] was a bit cumbersome, since a single detector had to be switched to each filter output in turn and the resulting data logged for later scaling and plotting. However, it represented a

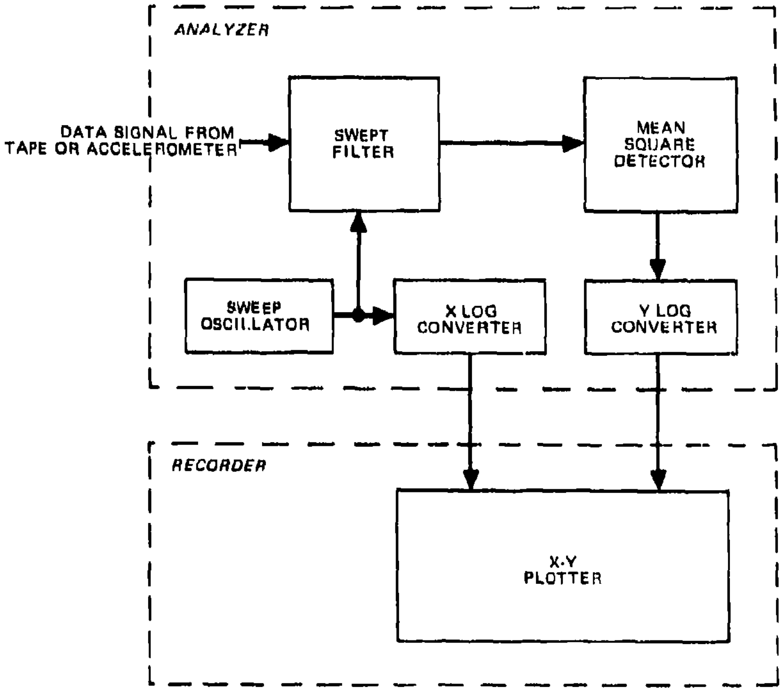

Fig. 6-6. Broadband random test data processing, swept-filter method.

considerable improvement over previous methods since it provided the capability for estimating amplitude distribution as well as spectral density. It was obviously impractical to use for on-line data processing.

To overcome the disadvantages noted above and to permit timely processing of large quantities of test data, the simultaneous detection of filter outputs and the automatic scaling and outputting of processed data are required. The method evolved in response to this need in the authors' laboratory is based upon use of a hybrid system [1]. The analog section contains 10-percent bandwidth filters (covering the nominal frequency range of 20 to 2650 Hz) and the amplifiers, etc., required for impedance matching, gain adjustment, and isolation. Its outputs are connected, via multiplexer and analog-to-digital (AD) convertor, to a small general purpose computer system which includes digital tape recorders and an incremental plotter. Signal detection is performed in the computer and the multiplexing and AD conversion rates are sufficiently high to allow effectively simultaneous detection. The computed power spectral densities are recorded on digital tape for subsequent outputting (listing or plotting) or further processing (see pp. 188-195). With a moderately large data volume, say 12 or more data

sequences, processing rates (raw data to report-quality plots) of six per hour are achieved easily.

If the frequency range of interest for data to be processed exceeds the range of a fixed-frequency filter system, the taped data can be played back at a different speed for processing. The relative speed change shifts the effective data frequency up or down (for increased or decreased speed, respectively) by the speed change ratio. However, it should be noted that an artificially changed signal analog sensitivity must be used if correct answers are to be obtained. This may be explained most easily by considering what occurs in a limited energy bandwidth on the tape record. For example, assume there exists in the data a bandwidth B of energy with uniform spectral density W and an rms value σ; then $\sigma^2 = WB$. If the tape is played back at twice the speed, it will be found that σ remains unchanged but, since the bandwidth is now $2B$, the computed spectral density would be $0.5W$ if no sensitivity adjustment were made. Therefore, computed spectral densities must be scaled by multiplying them by the ratio R of the record to playback speeds. This is equivalent to multiplying the original tape signal analog sensitivity by $\sqrt{R}$.

Special Purpose. On-line digital data processing is concomitant to the digital control method noted on page 160, and a hybrid technique is described in Refs. 110 and 111. Taped test data can be processed also by AD conversion and applying digital filtering or one of several transform algorithms. Various forms of digital analysis are possible: spectral, correlation, statistical, etc. [33,112,113]. Some of these types of analysis can also be accomplished using specialized analog or hybrid methods [114,115].

Swept or Stepped Narrowband Random Tests

On-line processing for data from these tests can consist of little more than some form of continuous graphic recording of detected control filter outputs and analogs of the filter tuning signals. Fig. 6-7 shows a typical setup for this purpose. For the stepped narrowband test, two additional options are available under the following conditions. If filter frequency-shift markers are recorded on the program tape, the detected outputs can be plotted on an xy plotter for each filter position in turn. Figure 6-8 diagrams a setup for doing so. The scanner output of the equalizing system can also be plotted (see Fig. 6-5) to document the spectral density outside the narrowband excitation.

The foregoing methods can be applied directly to taped data by playing the broadband signal back through the narrowband programming system, providing that all filter control signals and frequency-shift markers have been recorded.

Pulsed Excitation Tests

Definition of the vibration conditions during the pulsed excitation tests described on pp. 166-167 requires processing to determine the amplitudes of the fundamental frequency and a sufficient number of harmonics of the periodic signal for each of the prf's employed during test. The swept-filter analyzer

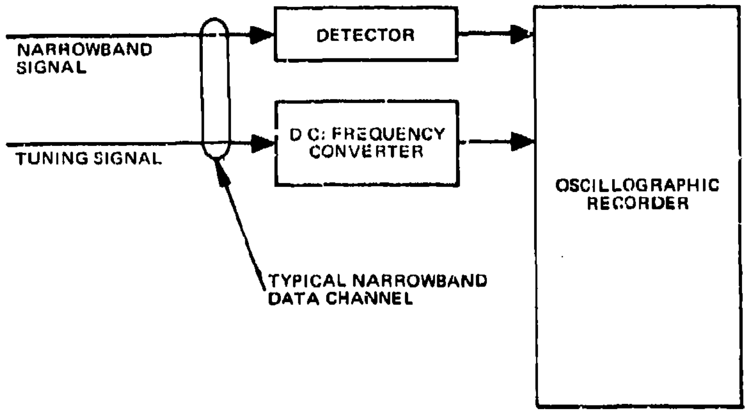

Fig. 6-7. Swept or stepped narrowband random test data processing, continuous.

system shown in Fig. 6-9 may be used to analyze a segment of the recorded signal at each prf by using tape loops. The amplitudes of the harmonics can be tabulated from the xy plot. If random vibration is also present, the xy plot can be used as an approximate measure of the acceleration spectral density in the frequency bands between the harmonics, providing allowance is made for the filter bandwidth characteristics.

The above method is time-consuming, i.e., expensive, and results in a large number of xy plots which, strictly speaking, should be converted to line spectra plots or tabulations. Again, a comb filter can be used to avoid the necessity of making tape loops and to obtain all the harmonics with one passage of the recorded data, provided only one harmonic of interest is present in the bandwidth of any filter. The hybrid analog-digital system described on page 185 can be modified for application to these test data, if certain supplemental information is recorded on the tape. The additional required data are the pulse-train excitation source and a prf code. The method provides for self-calibration of the processing system just prior to its use; i.e., the correct gain factors are calculated for each prf for the filters containing in their passbands the fundamental and the required harmonics. These calculations are performed by the computer and are possible because the ratio of pulse width to pulse period is maintained constant for all prf's; consequently, the relative amplitudes of pulse, fundamental frequencies and harmonics can be predetermined and entered into the computer program. The self-calibration eliminates any problems due to minor frequency shift or drift anywhere in the test control, recording, or playback sequences, since the gain factors are calculated for the actual playback data frequencies. Processed data are recorded on digital tape for subsequent outputting or further processing. The normal output is a tabulation of harmonic amplitudes for each prf as shown in Fig. 6-10.

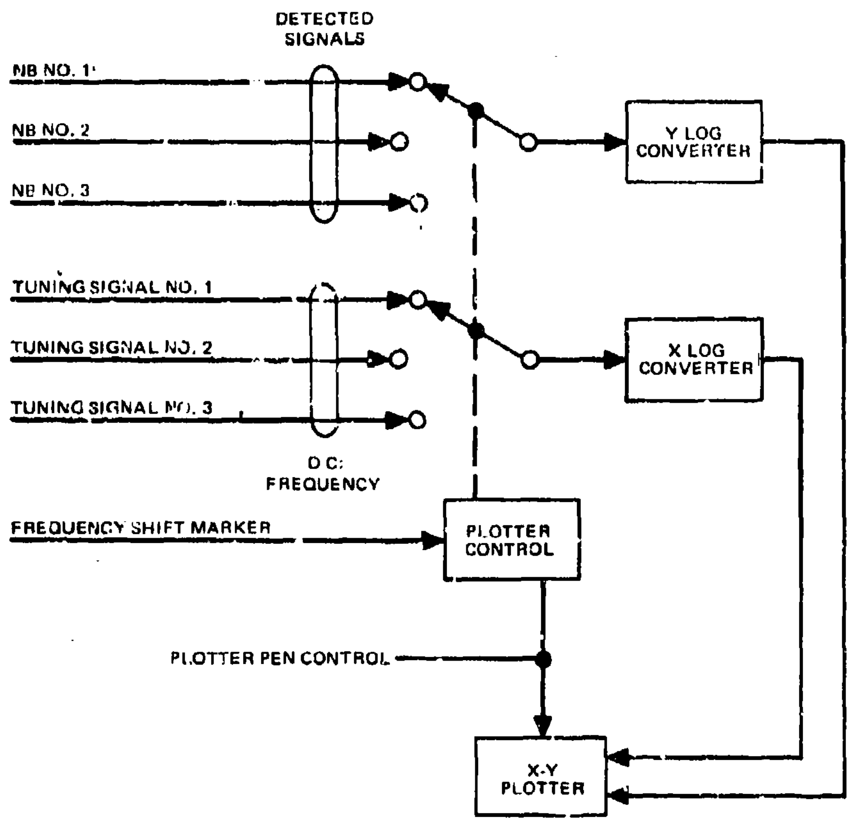

Fig. 6-8. Stepped narrowband random test data plotting

6.4 Special Topics

It was suggested in Chapter 2 that a common weakness in the design of vibration test experiments is the lack of consideration of requirements for the later analysis or evaluation of the data after initial processing, i.e., the cerebral processes. It is not unfair to suggest that the design of data analysis "systems" often displays a similar weakness in that the "system" stops at the point that the processed data are displayed on a cathode ray tube or an unlabeled xy plot. The material in this section describes some specialized processing applications of test data which have been found to be very useful. The descriptions are included to illustrate the potential of such specialized processing during the test evaluation phase. However, these methods depend on the availability of the results of initial processing on digital tape or hollerith cards and, of course, access to a digital computer. A data analysis system such as described on page 185 [1], which outputs data in this format, is particularly convenient. Bozich [116-118] has

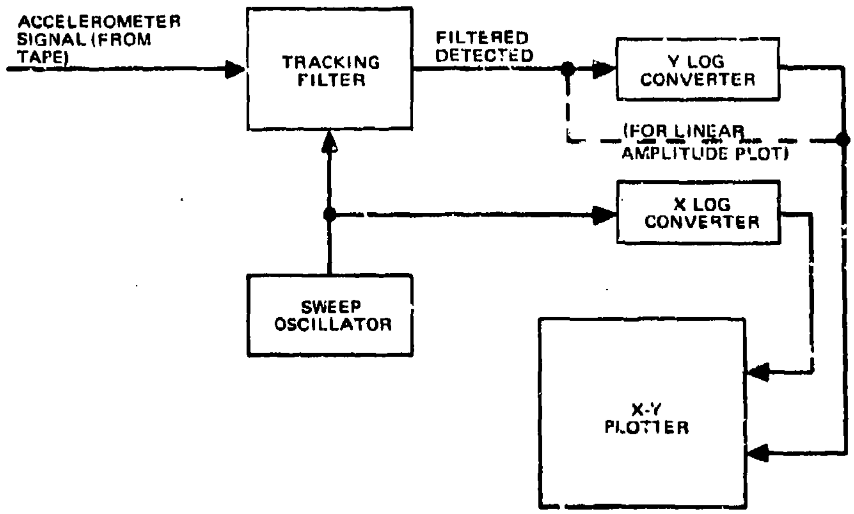

Fig. 6-9. Pulsed excitation test data processing, analog.

described similar data processing methods developed for test programs in which the sheer volume of test data required more efficient processing and evaluation of test data than has been traditional.

Random Test Data

The processed spectral records accumulated from one or more tests can be reprocessed in several useful ways after they are encoded on digital tape. Five basic types of routine data processing have been developed for use in the authors' laboratory; they are

1. Average and/or envelope of N spectra. Three different outputs are available: a plot of high, low, and average values (Fig. 6-11); a teletype (TTY) listing of high, low, and average values (Fig. 6-12); or a TTY listing of high and low values along with identification of the spectral records containing each (Fig. 6-13).

2. Ratio of two spectra. A sample plot for squared transmissibility is shown in Fig. 6-14. An option to plot the square root of the ratio, i.e., rms transmissibility, is available, as shown in Fig. 2-11.

3. Multiplying spectra by a constant or multiplying two spectra.

4. Computing the mean, standard deviation, and variance for N spectra. Fig. 6-15 shows a TTY listing for such computations for 350 spectra used to establish system gain and effective bandwidth constants.

5. Conversion of acceleration spectral density to, for example, displacement spectral density.

GUNFIRE VIBRATIØN DATA ANALYSIS

HIGH G.F. RATE REC. NØ. 0306 YEAR 1970

HARMØNICS: PEAK G'S BRØADBAND: RMS G'S

PRF'S

	107.6	106.2	104.8	103.4	102.1
FUND	.28804E+01	.26957E+01	.24425E+01	.24909E+01	.25139E+01
2ND	.46071E+01	.41001E+01	.35851E+01	.38037E+01	.39771E+01
3RD	.65996E+01	.57505E+01	.49076E+01	.51207E+01	.54002E+01
4TH	.42827E+01	.53876E+01	.52841E+01	.43163E+01	.47275E+01
5TH	.56532E+01	.59652E+01	.65630E+01	.55191E+01	.46730E+01
6TH	.66255E+01	.55587E+01	.56338E+01	.52530E+01	.47684E+01
7TH	.54909E+01	.60025E+01	.51158E+01	.60768E+01	.50187E+01
8TH	.24900E+01	.39180E+01	.37326E+01	.44991E+01	.54072E+01
9TH	.43561E+01	.35181E+01	.40837E+01	.48827E+01	.50876E+01
10TH	.21113E+01	.24279E+01	.42668E+01	.47021E+01	.52992E+01
BB-M	.11921E+02	.11847E+02	.11608E+02	.11795E+02	.11854E+02
BB-C	.10656E+02	.10544E+02	.10504E+02	.10648E+02	.10646E+02

PRF'S

	100.8	99.5	98.2	96.9	95.6
FUND	.24706E+01	.23853E+01	.29728E+01	.28973E+01	.26027E+01
2ND	.44866E+01	.45206E+01	.51757E+01	.49119E+01	.45743E+01
3RD	.58800E+01	.62748E+01	.63776E+01	.60254E+01	.62347E+01
4TH	.72606E+01	.57817E+01	.55403E+01	.52156E+01	.52417E+01
5TH	.35312E+01	.35690E+01	.42562E+01	.46560E+01	.54939E+01
6TH	.47614E+01	.49485E+01	.44107E+01	.44831E+01	.58666E+01
7TH	.48022E+01	.55859E+01	.48327E+01	.51977E+01	.58065E+01
8TH	.51691E+01	.53983E+01	.49162E+01	.54824E+01	.41555E+01
9TH	.41904E+01	.29141E+01	.36376E+01	.31672E+01	.19846E+01
10TH	.37174E+01	.44727E+01	.39994E+01	.34559E+01	.41944E+01
BB-M	.11863E+02	.11854E+02	.11862E+02	.11796E+02	.12024E+02
BB-C	.10721E+02	.10608E+02	.10521E+02	.10414E+02	.10752E+02

BB-M: MEASURED BRØADBAND (ALL HARMØNICS PLUS BASE NØISE)
BB-C: RMS ØF LINE SPECTRUM (10 HARMØNICS ØNLY)

Fig. 6-10. Typical teletype printout of gunfire vibration test data analysis.

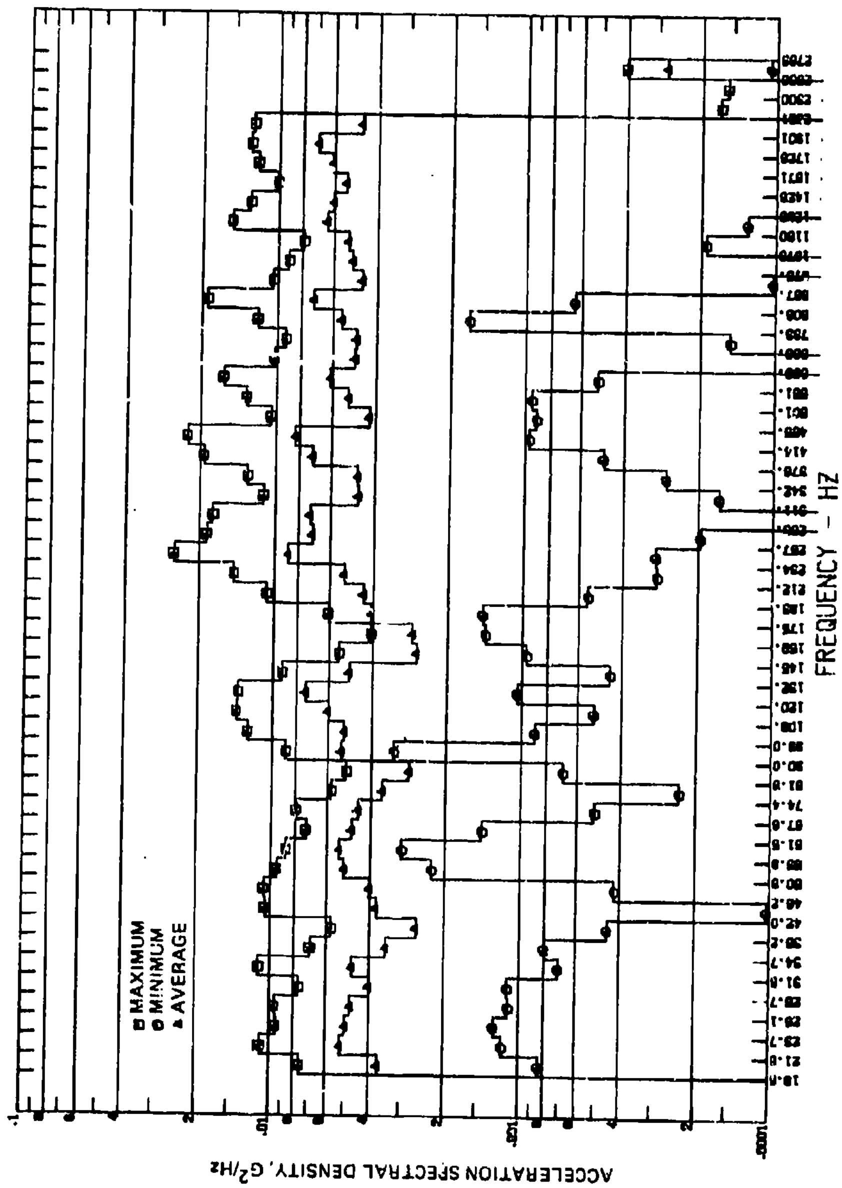

Fig. 6-11. Average and envelope of three spectra.

1-358		5000-69	5001-69	5002-69
CH.	FREQ.	HI	LØ	AVG
01	20.50	+.752901E-02	+.835057E-03	+.368897E-02
02	22.58	+.108769E-01	+.117285E-02	+.520239E-02
03	24.84	+.950529E-02	+.126444E-02	+.493144E-02
04	27.32	+.961030E-02	+.111084E-02	+.472821E-02
05	30.05	+.760128E-02	+.111900E-02	+.401541E-02
06	33.06	+.111201E-01	+.704126E-03	+.468905E-02
07	36.36	+.688798E-02	+.798545E-03	+.342050E-02
08	40.00	+.570138E-02	+.449430E-03	+.262237E-02
09	44.00	+.105242E-01	+.102683E-03	+.378919E-02
10	48.40	+.106422E-01	+.421366E-03	+.403611E-02
11	53.24	+.950735E-02	+.226168E-02	+.507278E-02
12	58.56	+.868498E-02	+.298090E-02	+.530376E-02
13	64.42	+.720609E-02	+.143201E-02	+.475475E-02
14	70.86	+.787595E-02	+.507508E-03	+.449178E-02
15	77.95	+.575931E-02	+.231635E-03	+.360446E-02
16	85.74	+.501468E-02	+.685613E-03	+.282137E-02
17	94.32	+.879970E-02	+.323544E-02	+.527734E-02
18	103.8	+.125154E-01	+.886212E-03	+.512838E-02
19	114.1	+.140522E-01	+.517052E-03	+.600143E-02
20	125.5	+.138413E-01	+.105142E-02	+.735354E-02
21	138.1	+.921740E-02	+.446606E-03	+.492721E-02
22	151.9	+.541886E-02	+.966392E-03	+.267880E-02
23	167.1	+.406588E-02	+.141514E-02	+.278212E-02
24	183.8	+.606444E-02	+.145722E-02	+.407941E-02
25	202.2	+.106896E-01	+.548996E-03	+.437784E-02
26	222.4	+.146496E-01	+.292180E-03	+.522333E-02
27	244.6	+.255888E-01	+.296837E-03	+.879485E-02
28	269.1	+.187286E-01	+.196458E-03	+.704189E-02
29	296.0	+.177919E-01	+.897333E-04	+.720997E-02
30	325.6	+.111368E-01	+.164588E-03	+.464924E-02
31	358.2	+.128891E-01	+.270308E-03	+.467141E-02
32	394.0	+.194381E-01	+.485997E-03	+.711046E-02
33	433.4	+.228109E-01	+.964986E-03	+.831464E-02
34	476.7	+.105556E-01	+.908979E-03	+.424589E-02
35	524.4	+.131850E-01	+.942351E-03	+.513402E-02
36	576.8	+.165241E-01	+.514266E-03	+.609933E-02
37	634.5	+.108923E-01	+.826527E-04	+.486085E-02
38	698.0	+.920019E-02	+.151387E-03	+.480263E-02
39	767.8	+.119731E-01	+.169685E-02	+.552667E-02
40	845.0	+.190883E-01	+.643282E-03	+.714202E-02
41	929.5	+.104943E-01	+.102812E-03	+.459519E-02
42	1022.	+.900267E-02	+.630089E-04	+.500402E-02
43	1124.	+.784342E-02	+.190337E-03	+.525239E-02
44	1236.	+.154188E-01	+.151349E-03	+.640348E-02
45	1360.	+.128883E-01	+.179918E-04	+.600843E-02
46	1496.	+.101087E-01	+.169822E-04	+.534670E-02
47	1646.	+.121012E-01	+.129297E-04	+.605922E-02
48	1811.	+.128717E-01	+.648383E-04	+.699744E-02
49	1992.	+.125999E-01	+.415562E-04	+.466160E-02
50	2191.	+.168690E-03	+.373388E-05	+.848497E-04
51	2410.	+.157504E-03	+.120689E-04	+.953734E-04
52	2651.	+.405349E-03	+.106621E-03	+.276671E-03

Fig. 6-12. Average and envalope of three spectra.

CH.	FREQ.	HI	LØ	HI NØ	LØNØ
01	20.50	+.752901E-02	+.835057E-03	1074	1076
02	22.58	+.108769E-01	+.117285E-02	1074	1076
03	24.84	+.950529E-02	+.126444E-02	1074	1076
04	27.32	+.961030E-02	+.111034E-02	1074	1076
05	30.05	+.760128E-02	+.111900E-02	1074	1076
06	33.06	+.111201E-01	+.704126E-03	1074	1076
07	36.36	+.688798E-02	+.798545E-03	1074	1076
08	40.00	+.570138E-02	+.449430E-03	1074	1076
09	44.00	+.105242E-01	+.102688E-03	1074	1076
10	48.40	+.106422E-01	+.421566E-03	1074	1075
11	53.24	+.930735E-02	+.226168E-02	1076	1075
12	58.56	+.868498E-02	+.298090E-02	1076	1075
13	64.42	+.720609E-02	+.143201E-02	1074	1075
14	70.86	+.787595E-02	+.507508E-03	1074	1075
15	77.95	+.573931E-02	+.231635E-03	1076	1075
16	85.74	+.501468E-02	+.683613E-03	1074	1075
17	94.32	+.879970E-02	+.323544E-02	1075	1076
18	103.8	+.125154E-01	+.886212E-03	1075	1074
19	114.1	+.140522E-01	+.517052E-03	1075	1076
20	125.5	+.138415E-01	+.105142E-02	1075	1076
21	138.1	+.921740E-02	+.446606E-03	1074	1076
22	151.9	+.541886E-02	+.966392E-03	1075	1076
23	167.1	+.406582E-02	+.141514E-02	1075	1074
24	183.8	+.606444E-02	+.145722E-02	1075	1074
25	202.2	+.106896E-01	+.548996E-03	1075	1074
26	222.4	+.146496E-01	+.292180E-03	1075	1076
27	244.6	+.255888E-01	+.296837E-03	1075	1076
28	269.1	+.187286E-01	+.196438E-03	1075	1076
29	296.0	+.177919E-01	+.897333E-04	1075	1076
30	325.6	+.111568E-01	+.164588E-03	1075	1076
31	358.2	+.128891E-01	+.270308E-03	1075	1076
32	394.0	+.194381E-01	+.465997E-03	1075	1076
33	433.4	+.228109E-01	+.964986E-03	1075	1076
34	476.7	+.105556E-01	+.902979E-03	1075	1074
35	524.4	+.131250E-01	+.942551E-03	1075	1074
36	576.8	+.165241E-01	+.514266E-03	1075	1074
37	634.5	+.102923E-01	+.826527E-04	1075	1074
38	698.0	+.920019E-02	+.151387E-03	1075	1074
39	767.8	+.119731E-01	+.169685E-02	1075	1074
40	845.0	+.190883E-01	+.643282E-03	1075	1074
41	929.5	+.104943E-01	+.102812E-03	1075	1074
42	1022.	+.900267E-02	+.630089E-04	1076	1074
43	1124.	+.784342E-02	+.190537E-03	1076	1074
44	1236.	+.154188E-01	+.131349E-03	1075	1074
45	1360.	+.126883E-01	+.179918E-04	1075	1074
46	1496.	+.101087E-01	+.169822E-04	1075	1074
47	1646.	+.121012E-01	+.129297E-04	1075	1074
48	1811.	+.128717E-01	+.648383E-04	1075	1074
49	1992.	+.125999E-01	+.415562E-04	1075	1074
50	2191.	+.168090E-03	+.373388E-05	1075	1074
51	2410.	+.157304E-03	+.129685E-04	1075	1074
52	2651.	+.405349E-03	+.106621E-03	1075	1074

Fig. 6-13. Envelope of three spectra and identification of source spectra.

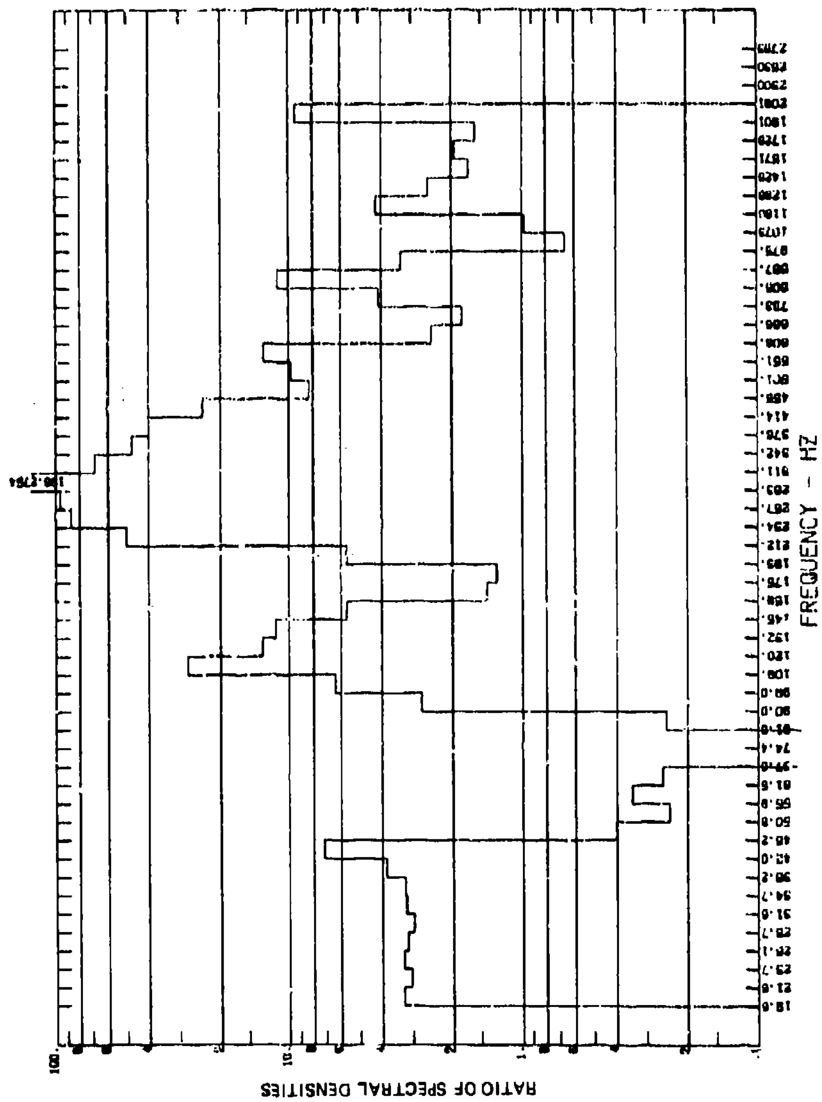

Fig. 6-14. Ratio of two spectra.

CH.	FREQ.	VA	SD	AVG
01	20.50	+.985193E-03	+.313877E-01	+.110300E+00
02	22.58	+.930341E-03	+.305014E-01	+.111810E+00
03	24.84	+.916479E-03	+.302733E-01	+.105880E+00
04	27.32	+.105808E-02	+.325281E-01	+.118691E+00
05	30.05	+.117489E-02	+.342767E-01	+.127314E+00
06	33.06	+.109930E-02	+.331557E-01	+.120721E+00
07	36.36	+.119949E-02	+.345336E-01	+.129850E+00
08	40.00	+.116158E-02	+.340819E-01	+.120165E+00
09	44.00	+.106915E-02	+.326979E-01	+.122704E+00
10	48.40	+.924028E-03	+.303978E-01	+.113475E+00
11	53.24	+.979003E-03	+.312890E-01	+.118641E+00
12	58.56	+.964532E-03	+.310569E-01	+.118151E+00
13	64.42	+.916577E-03	+.302750E-01	+.114316E+00
14	70.86	+.970079E-03	+.311461E-01	+.115925E+00
15	77.95	+.881220E-03	+.296853E-01	+.109451E+00
16	85.74	+.823372E-03	+.286944E-01	+.106793E+00
17	94.32	+.587514E-03	+.242387E-01	+.897558E-01
18	103.8	+.446516E-03	+.211309E-01	+.777253E-01
19	114.1	+.314026E-03	+.177208E-01	+.649369E-01
20	125.5	+.733031E-03	+.270745E-01	+.994476E-01
21	138.1	+.766269E-03	+.276815E-01	+.101[illegible]10E+00
22	151.9	+.741940E-03	+.272385E-01	+.997700E-01
23	167.1	+.748961E-03	+.273671E-01	+.101078E+00
24	183.9	+.790874E-03	+.281224E-01	+.103806E+00
25	202.2	+.795949E-03	+.282125E-01	+.104135E+00
26	222.4	+.792670E-03	+.281544E-01	+.104094E+00
27	244.6	+.639040E-03	+.252792E-01	+.931016E-01
28	269.1	+.740003E-03	+.272030E-01	+.997652E-01
29	296.0	+.698752E-03	+.264339E-01	+.976202E-01
30	325.6	+.699638E-03	+.264506E-01	+.978294E-01
31	358.2	+.698651E-03	+.264320E-01	+.976154E-01
32	394.0	+.669839E-03	+.258812E-01	+.957578E-01
33	433.4	+.670324E-03	+.258906E-01	+.956304E-01
34	476.7	+.629333E-03	+.250865E-01	+.925201E-01
35	524.4	+.606337E-03	+.246239E-01	+.907827E-01
36	576.8	+.625215E-03	+.250043E-01	+.924265E-01
37	634.5	+.618912E-03	+.248779E-01	+.916924E-01
38	698.0	+.559681E-03	+.236575E-01	+.873614E-01
39	767.8	+.552553E-03	+.235064E-01	+.865805E-01
40	845.0	+.538479E-03	+.232051E-01	+.853499E-01
41	929.5	+.501302E-03	+.223942E-01	+.823534E-01
42	1022.	+.576036E-03	+.240007E-01	+.884647E-01
43	1124.	+.586733E-03	+.242225E-01	+.894938E-01
44	1236.	+.616220E-03	+.248237E-01	+.913890E-01
45	1360.	+.581732E-03	+.241191E-01	+.887506E-01
46	1496.	+.571450E-03	+.239058E-01	+.880874E-01
47	1646.	+.674727E-03	+.259755E-01	+.955376E-01
48	1811.	+.668145E-03	+.258485E-01	+.947496E-01
49	1992.	+.639775E-03	+.252937E-01	+.921432E-01
50	2191.	+.622401E-03	+.249479E-01	+.905610E-01
51	2410.	+.607997E-03	+.246576E-01	+.893937E-01
52	2651.	+.913784E-03	+.285269E-01	+.103014E+00

NO. ØF SAMPLES = 350

Fig. 6-15. Mean, standard deviation and variance for 350 spectra

Response-Limited Tests

Efficient execution of the response-limited test described on pp. 167-168 depends on rapid computer processing of test data from each iterative step. Reference 1 describes the use of a computer to compare the response and control spectra, to calculate the input adjustments, and to output corresponding instructions to the test operator. Waiting time between test iterations has been reduced to about two hours for a test requiring control at 12 to 15 response locations. Traditional processing for such a test would require at least two days.

Deterministic Test Data

The processing methods illustrated above for random test data are equally applicable and desirable for deterministic test data, whether from a swept sinusoidal test [116] or from a pulsed excitation test. However, techniques for the conversion of such test data to digital form for evaluation are not presently well established. The applicability of these methods to the processing of data from modal tests and impedance measurements and for the comparison of analytical and experimental results is self-evident.

APPENDIX A
GLOSSARY OF ABBREVIATIONS AND SYMBOLS

AD	Analog to digital
b	Measure of slope of endurance (σ-N) curve
B,B′,BW	Bandwidth
c	Material constant, viscous damping coefficient
CRO	Cathode ray oscilloscope
D	Specific damping energy, damage coefficient
D_c	Damage coefficient for constant cyclic loading
D_0	Total energy dissipated
D_r	Damage coefficient for random loading
D_s	Damage coefficient for sweep frequency sinusoidal loading
e	Normalized standard error, 2.718 . . . .
E	Energy
E_c	Energy dissipated in sinusoidal dwell
E_r	Energy dissipated in random motion
E_s	Energy dissipated in sinusoidal sweep
f	Frequency, in hertzes
f_n	Natural frequency
$\lvert \dot{f} \rvert$	Absolute value of time rate of change of excitation frequency
F_i, F(t)	Forcing function
FM	Frequency modulation
g	Acceleration of gravity
G	Fraction of steady state response
h	Linear sweep constant
H	Amplification factor
H(iω)	Frequency response function
i	An index, $\sqrt{-1}$
IRIG	Inter-Range Instrumentation Group
J	Material constant
k	Spring constant
K	Arbitrary constant, fraction of steady state response
K_s	Proportionality constant between stress and vibration
m	Mass
n	Exponent of damping stress relationship, an index, number of channels
n_i	Number of cycles at stress level i
N	Number of cycles, number of occurrences
NSD	Loss-of-signal detector
p	An index denoting peak value

p( )	Probability density function
prf	Pulse-repetition frequency
P	Power
PSD	Power spectral density
P( )	Cumulative distribution function
q_n	Normal coordinate
Q	Peak amplification or quality factor
r	Frequency ratio (ω/ω_n); an index denoting random
rms	Root mean square
R	Response quantity, slope of spectral density curve
s(t)	Base motion coordinate
S/N	Signal-to-noise ratio
S_0	Peak displacement of excitation
$\ddot{S}_0$	Peak acceleration of excitation
S_e	Equivalent stress
SDF	Single degree of freedom
t	Time
T	Transmissibility, observation time
TDM	Time division multiplexer
T_c	Time of sinusoidal dwell
T_r	Time of random test
T_s	Time of sinusoidal sweep
TTY	Teletypewriter
U	Coordinate
VCO	Voltage-controlled oscillator
V_i	Input motion
V(f)	Velocity spectral density
VTVM	Vacuum tube voltmeter
W_0	Total strain energy
W(f)	Acceleration spectral density (g^2/Hz)
X(t)	Absolute motion coordinate
X(f)	Displacement spectral density
y(t)	Relative motion coordinate
$\overline{\dot{Y}^2}$	Mean square relative velocity
$\overline{\ddot{Y}^2}$	Mean square relative acceleration
Z_i	Input impedance
Z_0	Output impedance
β	Logarithmic sweep rate
ΔT	Increment of time
ζ	Fraction of critical damping
η	Sweep parameter
$\xi(t)$	Random time function
π	3.1415 . . .
σ	Stress level, rms level

τ	Time delay
ϕ_{in}	Eigenvalue
$\phi(t)$	Phase angle
ω	Circular frequency in rad/sec ($\omega = 2\pi f$)
ω_n	Natural frequency in rad/sec

APPENDIX B
USEFUL EQUATIONS AND RELATIONSHIPS

B.1 Root Mean Square of PSD

The following equations provide solutions for the root-mean-square (rms) magnitudes (acceleration, velocity, displacement) of acceleration spectral density spectra described by straight lines on log-log plots. The spectral density W is expressed in g^2/Hz, the frequency f in Hz, and the spectrum slope R in dB/octave, where

$$R = 3\ \frac{\log (W_2/W_1)}{\log (f_2/f_1)} .$$

RMS accelerations (g's):

$$rms = \left\{ \frac{3\, W_1\, f_1}{R + 3} \left[\left(\frac{f_2}{f_1} \right)^{(R+3)/3} - 1 \right] \right\}^{1/2}, \quad R \neq 3$$

$$rms = \left[W_1\, f_1 \ln \left(\frac{f_2}{f_1} \right) \right]^{1/2}, \quad R = -3$$

RMS velocity (in./sec):

$$rms = \left\{ \frac{11.32 \times 10^3\, W_1}{(R - 9)\, f_1} \left[\left(\frac{f_2}{f_1} \right)^{(R-3)/3} - 1 \right] \right\}^{1/2}, \quad R \neq -3$$

$$rms = \left[\frac{3774\, W_1}{f_1} \ln \left(\frac{f_2}{f_1} \right) \right]^{1/2}, \quad R = 3$$

RMS displacement (in.):

$$rms = \left\{ \frac{286.\text{l}\, W_1}{f_1^3} \left[\left(\frac{f_2}{f_1} \right)^{(R-9)/3} - 1 \right] \right\}^{1/2}, \quad R \neq 9$$

$$\text{rms} = \left[\frac{95.6\,W_1}{f_1^{\,3}} \ln\left(\frac{f_2}{f_1}\right)\right]^{1/2} \quad , \quad R = 9$$

For composite spectra made up of various straight-line segments, the total rms value is computed from the square root of the sum of the squares of the individual rms values.

B.2 Acceleration, Velocity, and Displacement Spectral Density Relationships

Equations for the conversion of spectral density values between displacement, velocity, and acceleration and between circular frequency ω (in rad/sec) and frequency f (Hz) are given below. A four-dimensional graph paper developed by Himmelblau [119] useful for such conversions is shown in Fig. B-1.

Displacement spectral density:

$$X(f) = 2\pi X(\omega) \qquad [\text{in.}^2/\text{Hz}]$$

Velocity spectral density:

$$V(f) = 4\pi^2 f^2 X(f) \qquad [(\text{in./sec})^2/\text{Hz}]$$

$$V(\omega) = \omega^2 X(\omega)$$

Acceleration spectral density:

$$W(f) = 2\pi\ W(\omega) \qquad [g^2/\text{Hz}]$$

$$W(f) = \frac{4\pi^2 f^2}{g^2} \qquad V(f) = \frac{16\pi^4 f^4}{g^2}\ X(f)$$

$$W(\omega) = \frac{\omega^2}{g^2} \qquad V(\omega) = \frac{\omega^4}{g^2}\ X(\omega)$$

B.3 Peak, Average, and RMS Relationships

Sinusoidal:

average absolute value = 0.636 peak value
rms value = 0.707 peak value
average absolute value = 0.9 rms value

Random:

$$\text{average absolute value} = 0.798 \text{ rms value}$$
(Zero mean value)

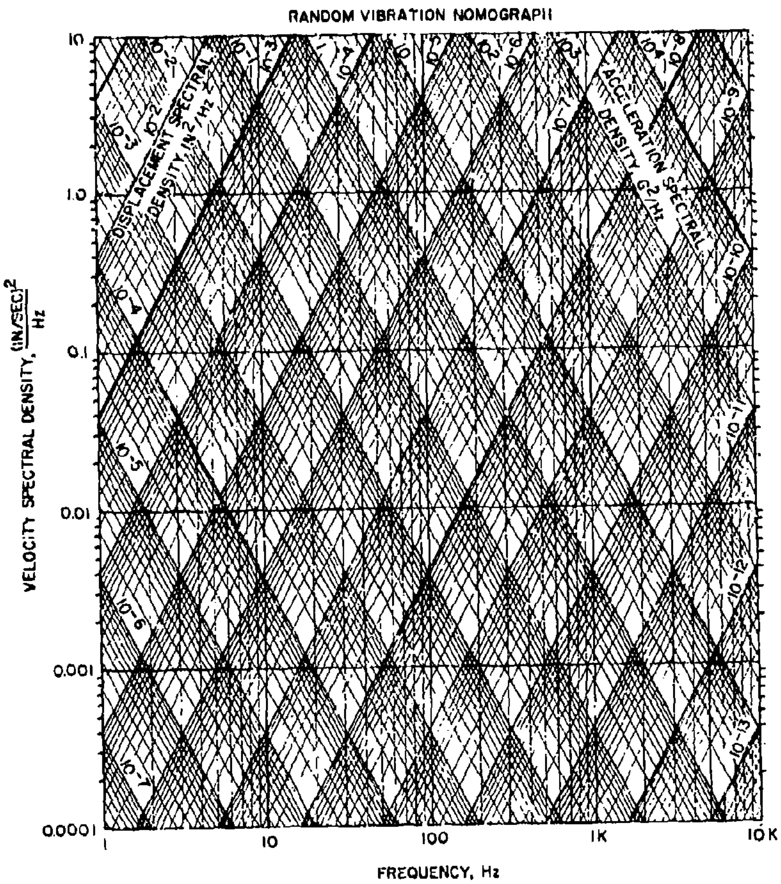

Fig. B-1. Universal random vibration nomograph.

B.4 Time for Sinusoidal Sweeps

Linear:

$$\text{Time} = \frac{f_2 - f_1}{h} \text{ (sec)}$$

h = sweep constant (Hz sec)

f = frequency (Hz)

Logarithmic: (See Fig. B-2)

$$\text{Time} = \frac{60}{B \ln 2} \ln\left(\frac{f_2}{f_1}\right) \text{ (sec)}$$

B = sweep rate (octaves/min)

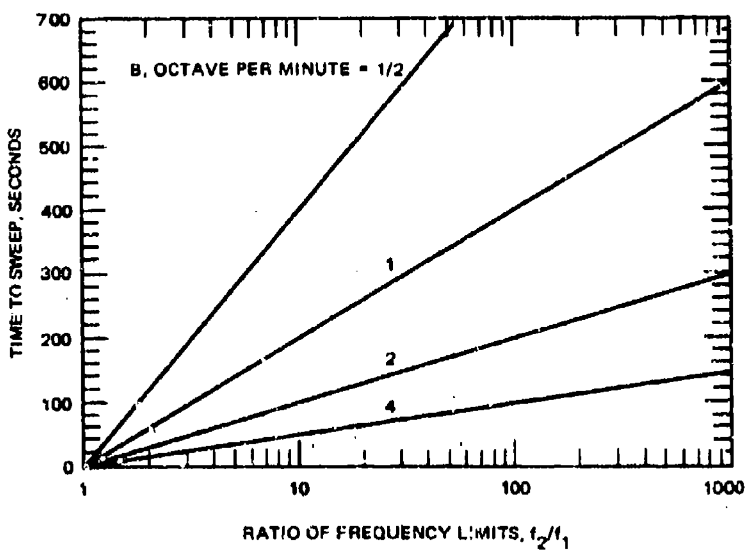

Fig. B-2. Nomograph for duration of logarithmic sinusoidal sweep.

APPENDIX C
VIBRATION TEST PLAN OUTLINE

It was suggested by reviewers of an early draft of this monograph that the inclusion of sample test plans or procedures and a checkoff list for selection of a test method would be helpful. The proper formats for test plans and procedures are often matters of strongly held personal opinion. Indeed, the differentiation between the two also may be an emotional topic. The authors believe that a test plan is a basic document which describes what is to be accomplished during a particular test program and, in broad terms, how it is to be carried out. On the other hand, a test procedure is a more detailed document, essentially in cook-book style, which describes very specifically the detailed steps to be employed to carry out the test. From these beliefs, and with some temerity, the following outline of a test plan was prepared to serve both as a model to facilitate preparation of a test plan and as a checkoff list by use of the references to various sections of the monograph in the column to the right of the outline.

The suggestion to include test procedures is believed inappropriate. Test procedures are very much a function of the equipment available at a particular test laboratory and of the general policies and practices established by the supervision of the laboratory and parent company management.

Appendix C
Test Plan Outline

1.	Objective	2.1
	Purpose, scope of test, data evaluation	
2.	General Requirements	
	2.1 Applicable Documents Listing of applicable equipment specifications, military specifications and standards, memoranda, etc. Applicable sections should be indicated.	
	2.2 Tolerances List of allowable tolerances for test and data reduction purposes	2.4
	2.3 Standard Conditions List of allowable laboratory conditions for demonstrating equipment functional performance	
	2.4 Test Documentation Requirements for content of test procedures, failure reporting, progress, and status and final reports	
	2.5 Failure Criteria Criteria for defining when a failure has occurred and how to proceed thereafter	2.1
3.	Test Program	
	3.1 Description of Test Item Physical and functional characteristics	2.1
	3.2 Test Fixtures Required characteristics	2.4 4.2
	3.3 Test Instrumentation Description of instrumentation characteristics—transducer types, locations, and mounting; signal conditioning; and recording requirements	4.3 6.1
	3.4 Test Facilities List of required facilities and their characteristics	4.0
	3.5 Test Conditions Description of test environment—level, duration, frequency range, control method, and location	2.2 4.3

3.6 Test Data — 2.3
Amount and characteristics of required data

3.7 Test Schedule
Usually shows approximate time spans of each phase of testing

4. Data Processing — 2.3
Description of data processing re... ents to convert recordings to reduced da... , PSD plots, etc. — 6.0

5. Data Evaluation — 6.0
Description of processing of reduced data for engineering evaluation

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